Unified Treatment for Liquid and Ice Particle Size Spectra Derived from the Stochastic Kinetic Equations

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The kinetic equation of stochastic condensation for cloud particle size spectra has been extended to account for crystalline clouds and also to include the accretion/aggregation process. The size spectra are separated into small- and large-size fractions that correspond to cloud drops (ice) and rain (snow). Solutions for the small-size fraction have the form of generalized gamma distributions and allow for bimodal ice cloud spectra. The general solution for the size spectra of the large-size particles is represented by the product of an exponential term and a term that is an algebraic function of radius. Simple analytical expressions are found for the particle size spectra that are functions of quantities available in cloud and climate models: liquid or ice water content and its vertical gradient, mean particle radius or concentration, and supersaturation or vertical velocities. The derived particle size spectra are compared with laboratory and field observations, as well as other simplified theoretical particle size distributions. These solutions provide explanations of the observed dependencies of the cloud particle spectra in different phases and size regimes on temperature, height, turbulence, vertical velocities, liquid or ice water content, and other cloud properties. These analytical solutions can be used for parameterization of the spectra of cloud and precipitating particles (both liquid and ice) and related quantities (e.g., optical properties and radar reflectivities) in bulk cloud microphysical parameterizations and in remote sensing techniques.

Copies of the papers: Khvorostyanov, Vitaly I., Curry, Judith A., 2008: Analytical Solutions to the Stochastic Kinetic Equation for Liquid and Ice Particle Size Spectra. Part I: Small-size fraction. Part II: Large-Size fraction n precipitating clouds. J. Atmos. Sci., In press, can be found online at: http://curry.eas.gatech.edu/onlinepapers.html

Small Size Particles

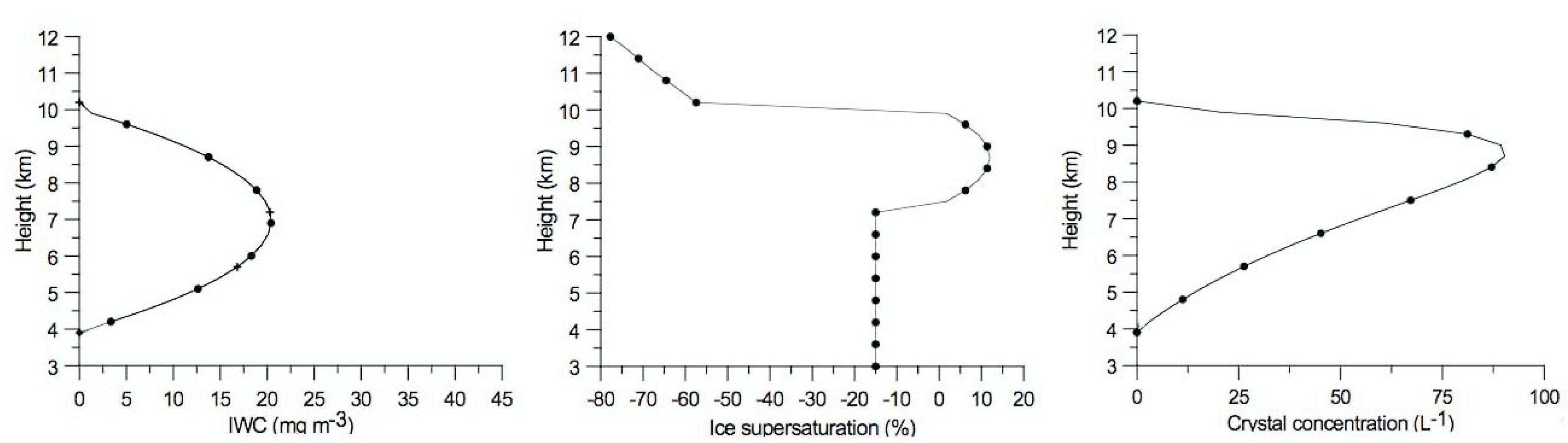
The general solution for the small-size fraction (where accretion rates are small and growth is dominated by condensation/deposition) have the form of generalized gamma distributions

$$f_s(r) = c_N r^p \exp(-\beta(r) r^{\lambda})\Phi(r)$$

The index p, which is expressed via supersaturation, cloud integral parameters (liquid or ice water content, mean radius or number density), and a few fundamental constants. Depending on supersaturation, the index p can be positive, yielding gamma distributions, or negative, yielding inverse power laws.

The solution differs from those found previously in the following ways. At small r, the solution is a power law with variable rather than fixed index p, allowing variable relative dispersions. The slope of the tail is not constant but varies with r; accounting for the greater depletion of large particles with greater sedimentation rate and includes depletion of the small fraction due to accumulation by the large fraction.

Input parameters for size spectra calculations



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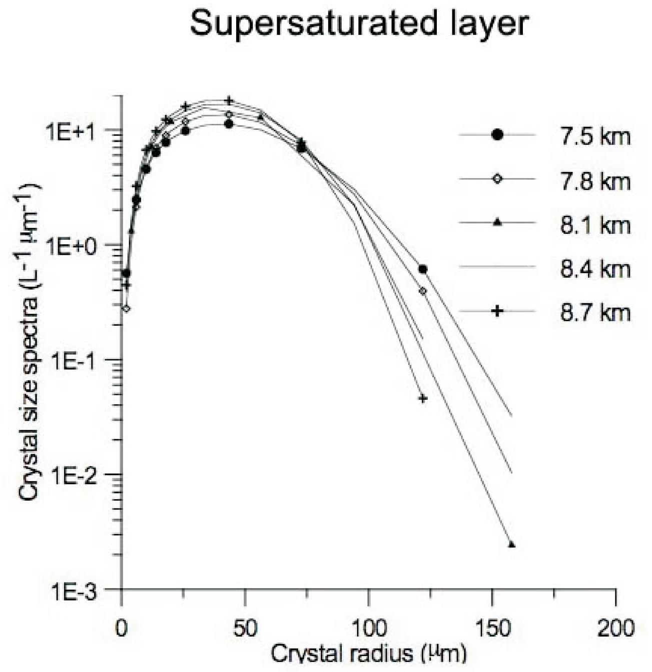

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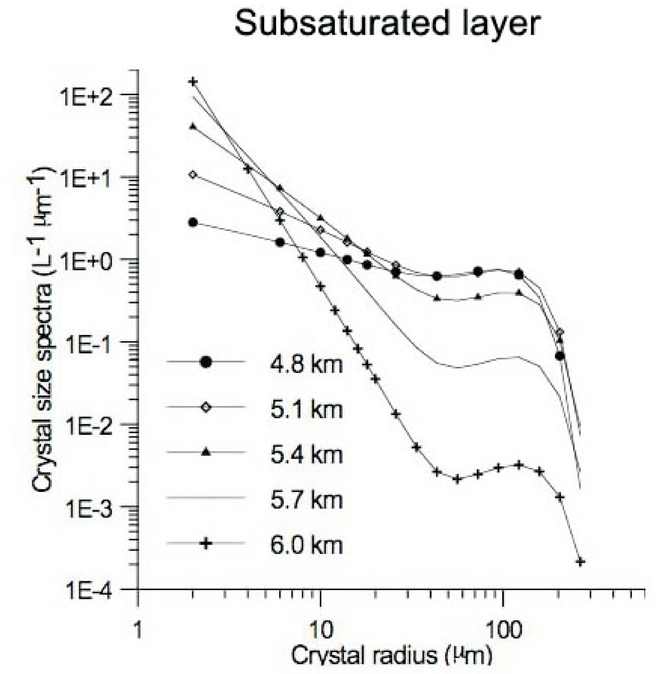


Large Size Particles

The general solution for the size spectra of the large-size particles (where growth by collection prevails) is represented by the product of an exponential term and a term that is an algebraic function of radius. Solutions are derived for three particular cases:

Fallspeed a linear function of particle size

$$f_l(r) = f_l(r_0)\left(\frac{r}{r_0}\right)^{p_l} \exp[-\beta_0(r - r_0)]$$

Neglect of condensation/evaporation

$$f(r) = f(r_0)\left(\frac{r}{r_0}\right)^{-B_v} \exp[-\beta(r)(r - r_0)]$$

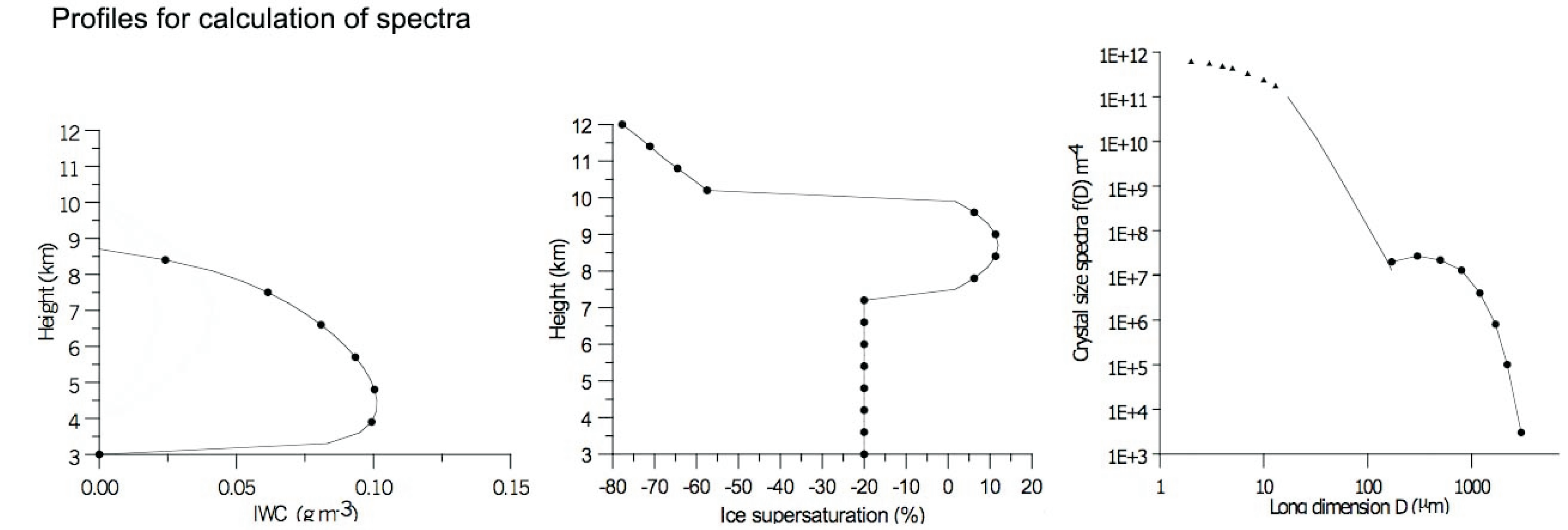
Fallspeed proportional to square root of particle size

$$f_l(r) \sim r^{-1/2} \exp[-\beta_{l0} r - V(r/H)^{1/2}]$$

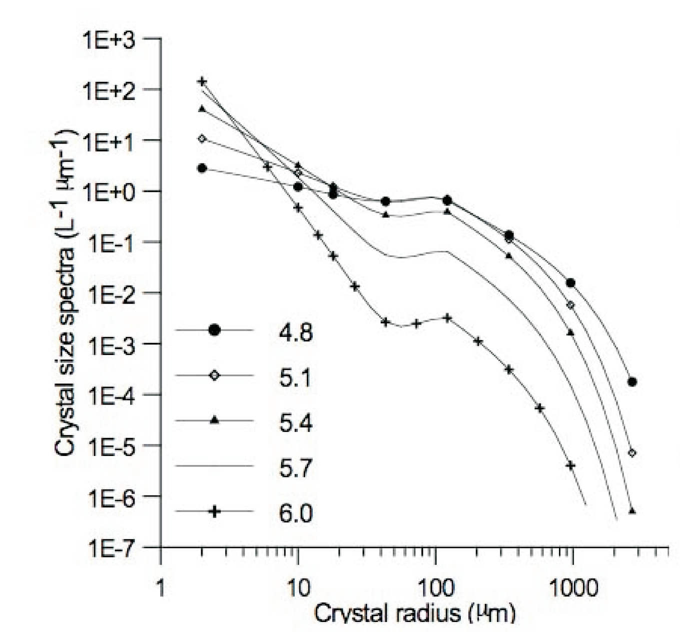
The terms in the analytical expressions are functions of quantities available in cloud and climate models: liquid or ice water content and its vertical gradient, mean particle radius or concentration, supersaturation, and vertical velocities.

These solutions provide explanations of the observed dependencies of the cloud particle spectra in different phases and size regimes on temperature, height, turbulence, vertical velocities, liquid or ice water content, and other cloud properties. These analytical solutions can be used for parameterization of the spectra of cloud and precipitating particles (both liquid and ice) and related quantities (e.g., optical properties and radar reflectivities) in bulk cloud microphysical parameterizations and in remote sensing techniques.

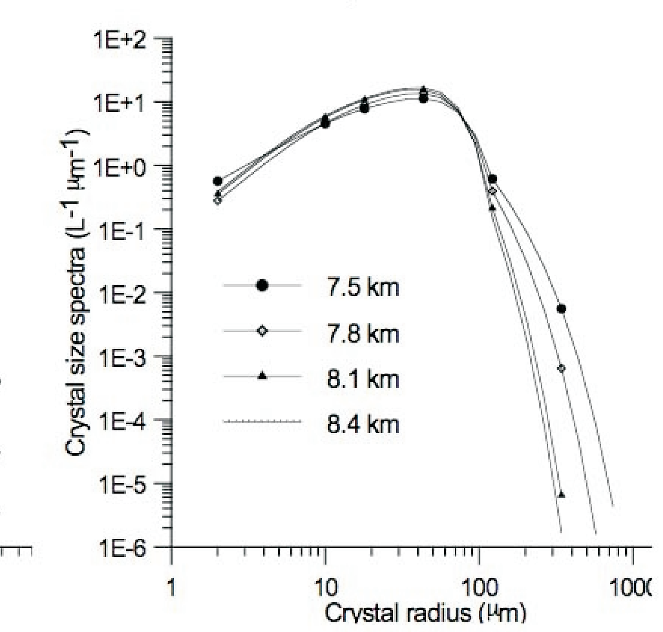
Profiles for calculation of spectra



Ice subsaturation



Ice supersaturation



Observed