Statistical Physics, Information Theory and Cloud Droplet Size Distributions

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Introduction

Specification of cloud droplet size distributions is essential for the calculation of radiation transfer in clouds and cloud-climate interactions, and for remote sensing of cloud properties. Despite the effort and progress made over the last few decades, a number of vital issues remain unsolved. For example, 1) It is well known that observed droplet size distributions are generally much broader than those predicted by the classical uniform model (Howell 1949). This so-called spectral broadening issue has puzzled many generations of cloud physicists. 2) An increasing amount of observational evidence has shown that the Weibull distribution best describes observed droplet size distributions (Liu et al. 1995; Costa et al. 2000; Liu and Daum 2000; Daum and Liu 2001). The physical reason for this is not clear. 3) It has been found that droplet size distributions depend on the scale over which they are sampled or simulated (Liu and Hallett 1998; Miles et al. 2000).

To answer these issues and develop ways to deal with cloud-related problems, a new formalism has been developed by integrating cloud physics with the ideas that have flourished in statistical physics and information theory (Liu et al. 1995, Liu 1995; Liu and Hallett 1997, 1998). This theory takes an approach different from traditional theories, and produces promising results. Here we review the theory, discuss major results derived from it, and explore its implications for treating clouds in climate models and addressing indirect aerosol effects.

The Systems Theory

Philosophy

The leitmotif of this application of systems theory is to obtain useful information on droplet size distributions without regard to the details of individual droplets. This is analogous to classic statistical mechanics where molecular systems are treated as a whole without invoking the details of individual molecule. It is assumed that a number of droplet size distributions may occur with different probabilities for a given droplet system. The current theory centers on two characteristic distributions: the most and the least probable droplet size distributions, denoted by MXSD and MNSD, respectively. When the averaging scale is large enough, observed size distributions are close to the MXSD. If the MNSD is identical with the MXSD, the clouds are expected to be uniform and the uniform model suffices. The equivalent microphysical model would be a uniform updraft with all droplets exposed to the same supersaturation and identical cloud condensation nuclei. However, such idealized situations never occur in nature. If there are differences between the MXSD and MNSD, individual size distributions will depend on the scale over which they are averaged. In this case, the comparability between theories and observations is questionable because of the potential scale-mismatch. Obviously, in the framework of the systems theory, the issues of spectral broadening, Weibull distribution, and scale-dependence are closely related. They are just different faces of the same coin.

Quantitative Formulation

Unlike the traditional theories mathematically formulated using differential equations, systems theory for droplet size distributions is built upon calculus of variations. Suppose we have a total amount of a variable, X (e.g., liquid water content) and N number of droplets. Clearly, there are a number of possible ways to distribute X among N; our goals are to find the MXSD and MNSD. The system can be represented by the following constraints (Liu and Hallett 1997):

$$\int \rho(x)dx = 1 \tag{1a}$$

$$\int x\rho(x)dx = \frac{X}{N} \tag{1b}$$

Where x is related to the physical processes controlling the droplet system; $\rho(x) = n(x)/N$ can be considered the probability that a droplet of x occurs, and n(x) is the droplet distribution with respect to x. By analogy with the Boltzmann entropy for molecular systems and the Shannon-Jaynes entropy generalized for complex multi-body systems, spectral entropy H is defined as

$$H = -\int \rho(x)\ln(\rho(x))dx \tag{2}$$

Maximizing Eq. (2), subject to the constraints described by Eqs. (1a) and (1b), we derive the MXSD to be the Weibull distribution

$$n_{max}(r) = N_0 r^{b-1} \exp(-\lambda r^b) \tag{3}$$

where the parameters $N_0 = ab/\beta$ and $\lambda = a/\beta$, and $\beta = X/N$. The general MNSD for this droplet system is

$$n_{min}(D) = N\delta(D - D_b) \tag{4a}$$

$$D_b = \left(\frac{\int r^b n(r)dr}{N}\right)^{1/b} = \left(\frac{X}{aN}\right)^{1/b} \tag{4b}$$

This is derived by maximizing the populational energy change (Liu and Hallett 1998)

$$E = -\frac{\pi\rho_w L}{6}\int D^3 n(D)dD + \pi\sigma\int D^2 n(D)dD + c \tag{5}$$

where the first term on the right side is the latent energy with L representing the latent heat of water; the second term is the surface energy with σ representing the surface tension of water. The coefficient c is related to the activated cloud condensation nuclei. The derivation of Eqs. (3) and (5) uses the power-law relationship between x and D

$$x = aD^b \tag{6}$$

where the parameters a and b are related to physical mechanisms controlling the droplet system. For the special case of liquid-water content conservation, $a = [1/(6\pi\rho_w)]$, and b = 3. The symbol ρ_w denotes the water density.

Weibull Distribution, Spectral Broadening, Scale-Dependence and Fluctuations

The above results for the MXSD and MNSD, as summarized in Figure 1, can be consistently used to explain all the three issues raised in the introduction. 1) For a given level of fluctuations, scale-dependence of droplet size distributions is anticipated because the MXSD is much broader than the MNSD. 2) Size distributions approach the MXSD as the averaging scale increases. A scale, defined as saturation scale, does not significantly change size distributions with further increases in averaging scale and are close to the Weibull distribution of the MXSD. 3) The change in size distributions is rapid and individual size distributions are ill-defined when averaging scales are less than the saturation scale. Spectral broadening may be due to scale-mismatches between observations and model simulations. 4) Given averaging scales, the difference between the MXSD and the MNSD and the saturation scale, increases with increasing fluctuations. Therefore, characterization of fluctuations, a subject poorly addressed so far, is crucial for cloud-related issues. It is noteworthy that real clouds are always in a more or less fluctuating state (Srivastava 1989). This suggests the extremely narrow size distributions such as the d-function will seldom be observed, no matter how uniform the cloud.

Implications

The new theory results have important implications for cloud representations in various numerical models and for studying indirect aerosol effects. It is common to couple large-scale dynamical models with microphysical models as detailed as allowed by computer resources. Such a direct coupling of models of different scales seems natural at first glance. However, such coupling is questionable because of the scale-dependence of individual droplet size distributions in turbulent clouds. For example, the MXSD should be used to represent droplet size distributions in climate models because of the large spatial and temporal scales involved. Similarly, size distributions measured by remote sensing

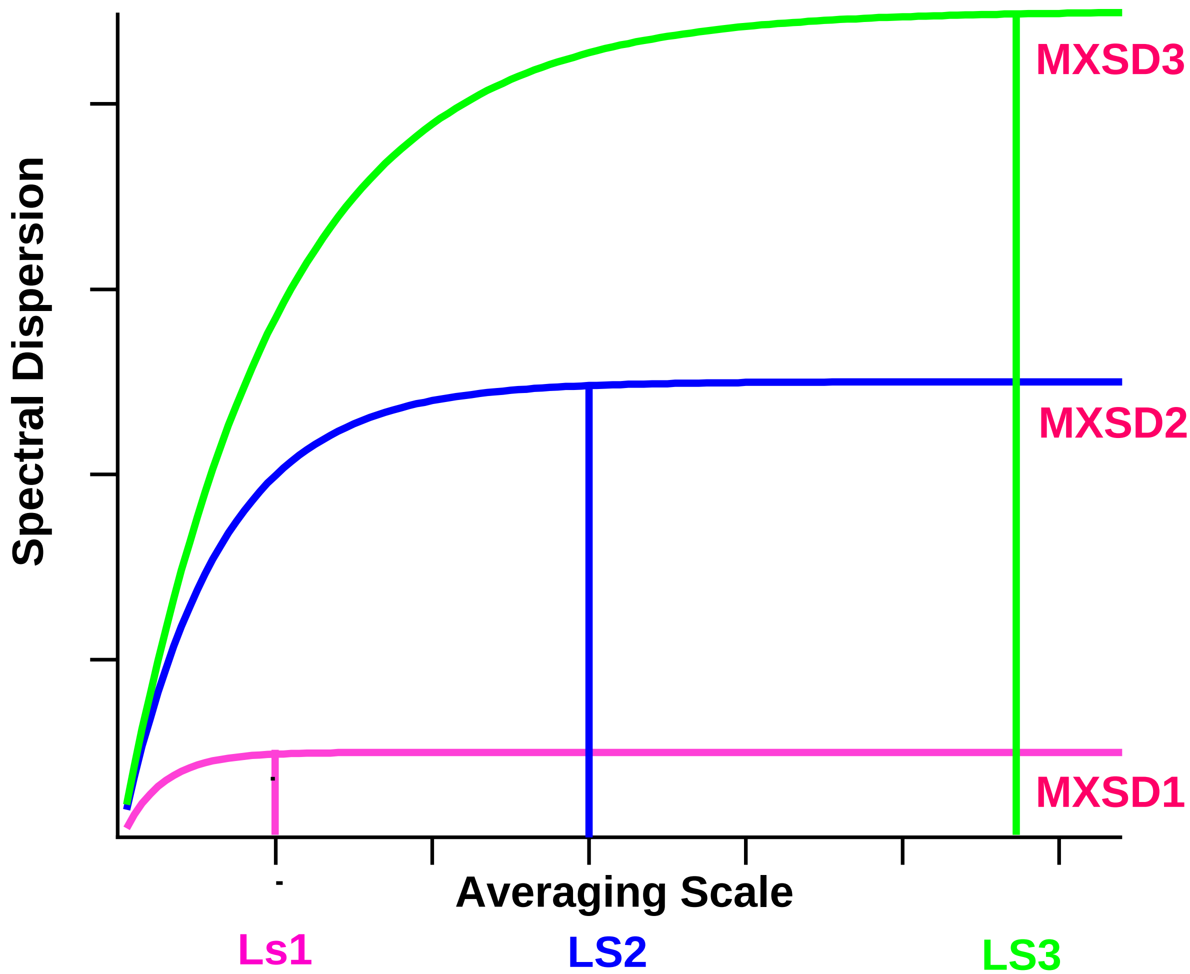

Figure 1. A diagram illustrating the scale-dependence of individual droplet size distributions at different levels of fluctuations. The meanings of both coordinates are only qualitative. The fluctuation increases from case 1 to case 3. The MXSD represent the maximum likelihood size distribution. This figure illustrates the following points addressed in the paper: 1) Observed size distributions approach the MXSD with the increase in averaging scales; 2) There exists a saturation scale beyond which droplet size distributions do not change much with further increases in averaging scales, denoted by L_s; 3) The stronger the fluctuation the larger the saturation scale ($L_{s3} > L_{s2} > L_{s1}$); 4) The spectral width of droplet size distributions increases with increases in averaging scales at a given level of fluctuations, and increases with increasing fluctuations at a given scale.

sensors such as radar and satellite, which typically sample large volumes, are expected to give distributions close to the MXSD. For models of smaller scales such as cloud-resolving models, it is not clear whether the scales involved are large enough to meet the requirement of scale saturation. When the averaging scales involved are less than the saturation scale, treating droplet size distributions becomes more complicated. Size distributions are ill-defined without specifying the scale involved. This unique property suggests the necessity of making explicit references to the averaging scale in numerical models. Actually, the need for a paradigm-shift from a scale-independent to a scale-dependent theoretical framework is emerging in many fields where a variety of fluctuations and scales are involved. Before scale-dependent models become a reality, considering variability at sub-grid scales is important. Finally, the vital role of fluctuations in determining droplet size distributions demands better understanding and consideration of cloud turbulence and the effects of variation in cloud condensation nuclei (CCN) properties. As demonstrated in Daum and Liu (2001), the increase of spectral dispersion (decrease in b), caused by either turbulence or the influences of anthropogenic aerosols, will increase cloud effective radius, decrease cloud albedo, and exert warming effects on climate.

Concluding Remarks

In the quest to understand and explain observed droplet size distributions, major efforts have been devoted to various kinetic models built upon differential equations; the idea of the systems theory has received much less attention. At first glance, the systems theory probably seems to convey a "strange" impression that observed size distributions have little to do with the details of individual droplets and their interactions. It is interesting that physicists shared a similar impression on statistical mechanics during the early days of this discipline. However, such a view was refuted by the later success of statistical mechanics. The ideas of statistical mechanics have been successfully extended to various disciplines to study complex systems (Haken 1977). Such widespread successes provide indirect justification for using the systems approach to study cloud droplet size distributions. The new theory has also been justified by its successful explanations for many observed phenomena such as spectral broadening and scale-dependence. It may serve as a new avenue to some key issues concerned by Atmospheric Radiation Measurement (ARM) program.

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