Stochastic Radiative Transfer in Broken Clouds: Validation Tests

E. Kassianov, T. P. Ackerman, R. T. Marchand, and M. Ovtchinnikov
Pacific Northwest National Laboratory
Richland, Washington

Introduction

An approach for the stochastic description of the solar radiation transfer through broken fields with the arbitrary horizontal and vertical inhomogeneity have been introduced (Kassianov 2000). Different combinations of the random and maximum cloud overlap can be treated by the suggested approach. We derived the approximated equations for both the mean direct and diffuse solar radiance on the basis of the stochastic transfer equation and a new statistically inhomogeneous Makovian model of broken clouds. In this paper we estimate the accuracy and robustness of the approximated equations by using three-dimensional (3D) broken cloud fields that were (1) produced by the Boolean stochastic model, (2) simulated by a large-eddy simulation (LES) model and (3) derived from collocated and coincident Multi-angle Imaging SpectroRadiometer (MISR) and ground-based observations.

Markovian Approach

In this section, an overview of the suggested approach is given. The irregular geometry of broken clouds has been described by unconditional $\langle\kappa(r)\rangle = P\{\kappa(r) = 1\}$ and conditional $V(r_2, r_1) = P\{\kappa(r_2) = 1 \mid \kappa(r_1) = 1\}$ probabilities of the cloud presence (Markovian assumption). Here $\kappa(r)$ is the random indicator field, $\kappa(r) = 1$ inside clouds, and $\kappa(r) = 0$ outside clouds, $r = (x,y,z)$, the angular brackets will be used for ensemble averages over $\kappa(r)$. A new statistically inhomogeneous Markovian model has been suggested to describe both the horizontal and vertical variability of broken clouds. The term "statistical inhomogeneity" is understood to mean that the unconditional probability (or cloud fraction) $\langle\kappa(r)\rangle$ depends on the vertical coordinate and the conditional probability $V(r_2,r_1)$ depends on the mutual arrangement of the points r_2 and r_1. As an illustration, some examples are given below:

If the point $r_1 = (x_1, y_1, z_1)$ and $r_2 = (x_2, y_2, z_1)$ belong to the same kth layer, then

$$V_k(r_2, r_1) = (1 - p_k) \times \exp(-A_k \times |r_2 - r_1|) + p_k \tag{1}$$

where p_k is the cloud fraction in the kth layer, and A_k is the parameter;

If the point $r_1 = (x_1, y_1, z_1)$ and $r_2 = (x_1, y_1, z_2)$ belong to different adjacent layers, namely the kth and mth layers, then

$$V_{k,m}(r_2, r_1) = \exp\left(-A^* \times |r_* - r_2|\right) \times \{V_k(r_1, r_*) - p_m\} + p_m, \tag{2}$$

where $r_* = r_1 + \omega \times (z_* - z_1)/c$, $\omega = (a, b, c) = (r_2 - r_1)/|r_2 - r_1|$.

For the upward direction (c>0), m=k+1, $A^* = A_{k-1}^{up}$ and z_* equals the top altitude of the kth layer, $z_* = z_{k+1}$; for the downward direction (c<0), m=k-1, $A^* = A_{k+1}^{dw}$ and z_* equals the base altitude of the kth layer, $z_* = z_{k-1}$. Parameters A_k^{up} and A_k^{dw} determine the statistical relationship between kth layer and its upper (k+1) and lower (k-1) adjacent layers, respectively. Note, all these parameters A_k, A_k^{up} and A_k^{dw} depend on both the 3D cloud structure and the positions of points r_1 and r_2. By changing the values of these parameters, one can describe different combinations of maximum and random cloud overlaps. This sketched flexibility of the inhomogeneous model is its appealing feature.

The statistically inhomogeneous model of broken clouds and the stochastic transfer equation were used to derive approximated equations for the mean solar radiance. It was assumed that for each kth layer the domain-averaged optical properties are constant (piecewise constant approximation), e.g., the extinction coefficient $\sigma(r) = \sigma(z) = \sigma_k$, the single scattering albedo $\omega_0(r) = \omega_0(z) = \omega_{0,k}$ and the scattering phase function $g(r, \omega, \omega') = g(z, \omega, \omega') = g_k(\omega, \omega')$. Also it was assumed that a parallel unit flux of solar radiation is incident on the upper boundary of a given cloud field in direction $\omega_\oplus$. To get the absolute values, the calculated radiative properties should be multiplied by the spectral solar constant weighted with $\cos(\xi_\oplus)$, where $\xi_\oplus$ is the solar zenith angle (SZA).

The equation for the mean solar radiance can be written as

$$\langle I(z, \omega)\rangle = \frac{1}{|c|} \int_{E_z} \omega_0(\xi)\phi(z, \xi)d\xi \int_{4\pi} g(\xi, \omega, \omega') f(\xi, \omega') d\omega' + \langle j(z, \omega)\rangle \delta(\omega - \omega_\oplus) \tag{3}$$

where $E_z = (hb, z)$ if c>0, and $E_z = (z, ht)$ if c<0, ht and hb are the top height and the base height of cloud field, respectively; $\langle j(z, \omega)\rangle$ is the mean direct (unscattered) radiance, function, and $f(z, \omega) = \sigma(r)\langle\kappa(r)I(r, \omega)\rangle$ is the mean collision density, $\delta(\cdot)$ is Dirac's delta function.

An integral equation for the mean collision density has the form

$$f(x) = \int_X k(x, x') f(x') dx' + \Psi(x) \tag{4}$$

$$k(x, x') = \frac{\omega_0(z) g(z, \omega, \omega') \eta(r, r')}{2\pi |r - r'|^2} \delta\left(\frac{r - r'}{|r - r'|} - \omega\right) \tag{5}$$

$$\Psi(x) = \sigma(z)p(z)\,\nu(z,\omega)\,\delta(\omega - \omega_{\oplus}) \tag{6}$$

where X is the phase space of coordinates and directions, $x = (r, \omega)$. All functions $\phi(r, r')$, $\eta(r, r')$ and $\nu(z,\omega) = \langle \kappa(r) j(r,\omega) \rangle / p(z)$ are defined by the recurrente expressions.

The closed system of Eqs. (3) - (4) can be solved by using any appropriate numerical methods or approximated ones (e.g., the spherical harmonic method). Here, to solve these approximated equations, we apply the Monte Carlo method. The latter is based on simulating a Markovian chain (see, e.g., Marchuk et al. 1980). In particular, we used the method of direct simulation to calculate the absorption and radiative fluxes. n this simulation technique, the photon trajectory modeling was made in correspondence with the initial $\Psi(x)$ and transitional $k(x, x')/\omega_0(z)$ densities of integral Eq. (4), while the radiative properties were estimated in accordance with their physical contents (Marchuk et al. 1980).

The statistically inhomogeneous model has relatively few input parameters, which describe only the bulk geometrical statistics of the 3D broken cloud field. Thus the question arises: How accurately can equations derived on the basis of the statistically inhomogeneous model represent the mean radiative properties of the 3D broken cloud fields (e.g., mean fluxes, mean absorption)? To answer this question, the following steps are taken for different 3D cloud fields. First, we calculate the mean radiative properties exactly by using a given full 3D cloud structure. The obtained radiative properties will be considered as a reference. Second, for a given 3D cloud field, we calculate the bulk cloud statistics, that will be served as input data for the statistically inhomogeneous model. Next, we estimate the mean radiative properties by applying the approximated Eqs. (3) and (4). The obtained radiative properties will be considered as approximations of true ones. Finally, we compare the mean radiative properties obtained by using the independent exact method (references) with ones obtained for the statistically inhomogeneous model (approximations).

Cloud Fields

In our analysis we used three different fields of 3D broken clouds, which represent marine low-level cumulus clouds. For each of these fields, the bulk geometrical statistics have were derived and then used as input data to calculate the mean radiative properties (next section).

Boolean cloud fields. An ensemble of cloud realizations (Figures 1a and 1d) was obtained by using a Boolean stochastic model (see, e.g., Stoyan et al. 1995). Note, Boolean models can be considered as fundamental models for stochastic geometry, and they allow one to easily obtain an ensemble of cloud field realizations (samples) with given mean geometrical properties (e.g., cloud fraction, horizontal *D* and vertical *H* sizes) and different vertical structure. In our simulations, the values of these parameters were matched to the typical values for marine low-level cumulus clouds. Below we use the term "Boolean cloud field" as a reference to cloud field(s) obtained on the basis of the Boolean model.

LES cloud field. The second field of marine low-level cumulus clouds (single realization) was provided by the LES model (Figures 1b and 1e). Data from the Atmospheric Radiation Measurement (ARM) Program's Tropical Western Pacific (TWP) site was used to initialize the LES model. The cloud field was simulated in a domain $10\times10\times2$ km^3 with a 0.1–km horizontal and 0.033-km vertical resolution. The simulated 3D cloud field is highly variable in both horizontal and vertical dimensions. For example, the height of the cloud base above the lifting condensation level varies in the interval from 0.1 km to 0.6 km, while its mean value is equal to 0.2 km. Below we use the term "LES cloud field" to represent the cloud field derived from LES simulations.

MISR cloud field. The third field of marine low-level cumulus clouds (single realization) was obtained from collocated and coincident MISR and ground-based radar observations in the TWP region at the island of Nauru (Kassianov et al. 2001). The reconstructed 3D geometry of broken clouds (Figures 1c and 1f) corresponds to the domain $\sim30\times30\times2$ km^3 with a 0.275–km horizontal and 0.045-km vertical resolution (total number of pixels is $110\times110\times44$). We use the term "MISR cloud field" to correspond to the cloud field derived from MISR retrieval.

Simulations of Boolean Fields

The Boolean cloud fields were used to estimate the accuracy of the approximated Eqs. (3) and (4). The latter were obtained for Markovian cloud fields. To estimate the accuracy correctly, we simulated Markovian cloud fields. In particular, the spatial Poisson process was simulated by using the Boolean model (see, e.g., Stoyan et al. 1995). We assume that broken clouds occupy a certain region in space (simulation domain) in the form of a parallelepiped with thickness H, the base of which is a square with sides xl. In our simulations, the 3D cloud fields (cloud realization) were formed by a group of truncated paraboloids of revolution with a fixed diameter D and height H (aspect ratio $\gamma = H/D$). The values of these parameters, which are typical for small marine cumulus clouds (see, e.g., Benner and Curry 1998), have been used in our simulations. We divided this entire region into identical $100\times100\times10$ pixels. Each pixel has the same horizontal $\Delta x = \Delta y$ =0.1 km (xl=10 km) and vertical Δz =0.045 km sizes. Note that the vertical resolution of radar observations is 0.045 km (see, e.g., Clothiaux et al. 1999). Then the value of the indicator field $\kappa(r)$ is determined for each pixel (*i*, *j*, *k*) as a value of $\kappa(r)$ at a point $r^* = (x_i, y_j, z_k)$, where $x_i = i\Delta x$, $y_j = j\Delta y$, $z_k = k\Delta z$, $i = 1,\ldots,100$, $j = 1,\ldots,100$, and $k = 1,\ldots,10$. The pixel (i, j, k) is considered as a cloudy pixel if $\kappa(r^*) = 1$ (point r^* belongs to the cloud). The reverse is true if $\kappa(r^*) = 0$ (point r^* does not belong to the cloud).

Bulk Cloud Statistics

There are a few basic steps for deriving bulk geometrical statistics for a given 3D cloud field. First, we calculate the cloud fraction (the unconditional probability of cloud presence) for each layer of the 3D cloud field. Second, we compute the conditional probabilities of cloud presence for each kth layer in both the x- and y-directions. Third, we determine the conditional probabilities of cloud presence for two adjacent layers in both the upward and downward-directions. Finally, we derive parameters $A_{k,x}$ (x-direction), $A_{k,y}$ (y-direction) and A_k^{up}, A_k^{dw} (upward and downward-directions) for each kth layer

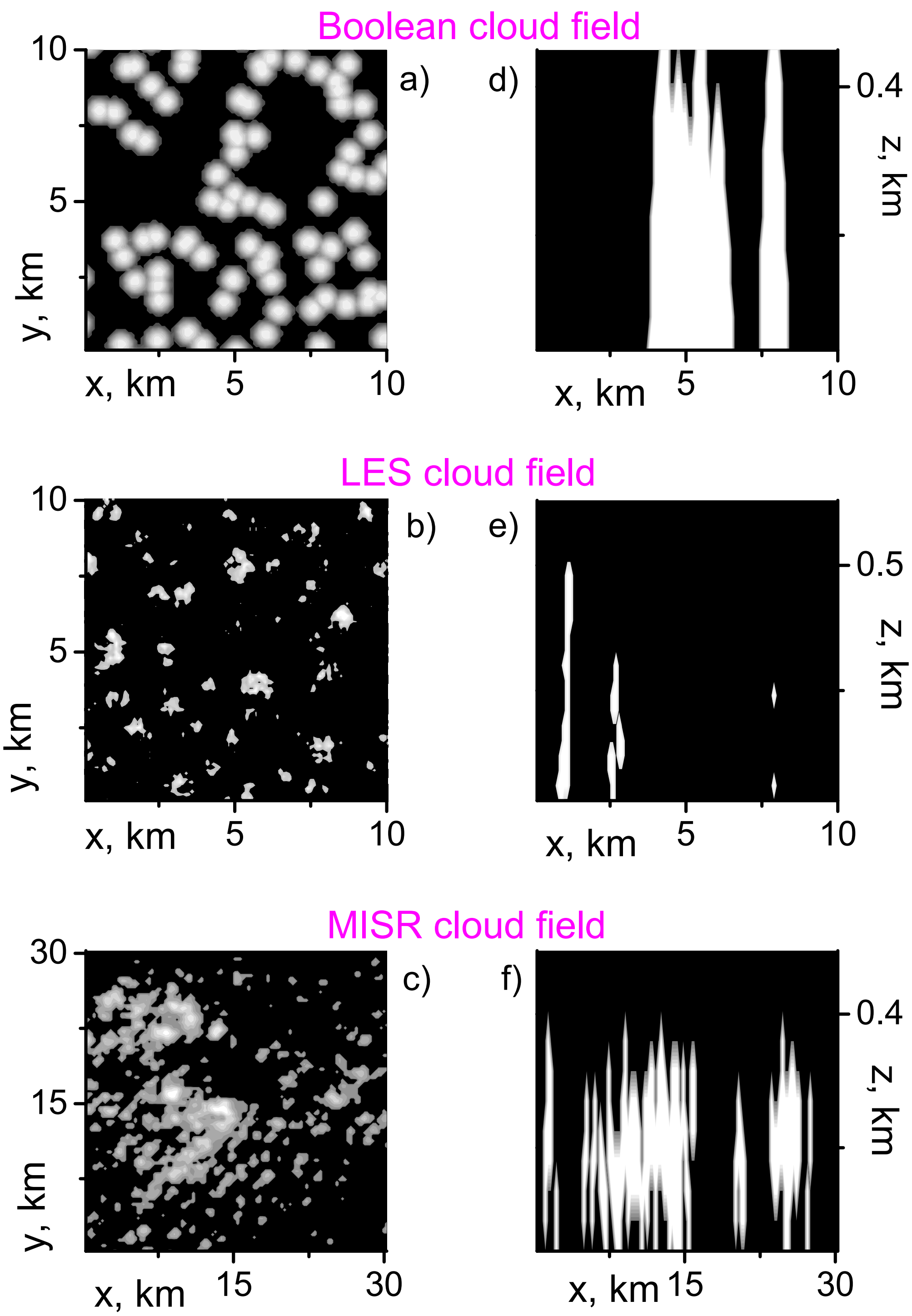

Figure 1. The horizontal (left column) and the vertical (right column) distributions of broken clouds that are provided by the Boolean stochastic model Figures a) and d), LES model Figures b) and e) and MISR cloud retrieval Figures c) and f). The vertical cross sections corresponding to y = y1/2, where y1 is domain size in y-direction. Brightness in the horizontal distributions (left column) is proportional to the geometrical thickness of clouds.

by using obtained probabilities and Eqs. (1) and (2). These probabilities have been derived from a given 3D cloud field. If two adjacent cloud layers, namely kth and (k-1)th layers, are perfectly dependent (maximum cloud overlap), then $V_{k,k-1}(r_2, r_1) = 1$ and $A_k^{dw} = 0$. If these two adjacent cloud layers are perfectly independent (random cloud overlap), then $V_{k,k-1}(r_2, r_1) = p_{k-1}$ and $A_k^{dw} >> 1$. In a similar way, parameters A_k^{dw} and A_k^{up} can be calculated for different positions of points r_1 and r_2.

The cloud statistics can be calculated from the single cloud realization (domain-averaged values) or from an ensemble of cloud realizations (ensemble-averaged values). For both the MISR cloud field and the LES cloud field, only domain-averaged cloud statistics were derived from corresponding single cloud realizations. For the Boolean model, the ensemble-averaged cloud statistics were obtained (averaging over 10000 realizations). We performed simulations of the Boolean cloud fields and corresponding radiative calculations for a set of cloud geometrical parameters, but here we present results for mean cloud diameter D=1 km, cloud height H=0.5 km (aspect ratio $\gamma = 0.5$), and the nadir-view cloud fraction $N_{nadir} = 0.5$. The numerical experiments with other cloud geometrical parameters provide similar results (accuracy of approximated equations), which are discussed below. The examples of the bulk cloud statistics are presented in Figure 2.

Radiative Calculation Results

Boolean Cloud Fields

To estimate the accuracy of the suggested statistical approach we compare the ensemble-averaged radiative properties obtained by two independent methods:

Numerical averaging method. By applying the Boolean stochastic model (see previous section), realizations of the Markovian cloud field $\kappa(r)$ are simulated for a set of cloud parameters (e.g., nadir-view cloud fraction, the mean horizontal size). In each of these 3D realizations we calculated radiative properties by using a Monte Carlo method (the maximum cross section approach) and periodical boundary conditions. The ensemble-averaging radiative properties were obtained after appropriate processing. Since the full 3D cloud geometry is used in the radiative calculations, the obtained mean radiative properties are considered as references. To denote these reference radiative properties, we will use subscript "ref".

Analytical averaging method. Approximated equations for the mean radiance, which have been obtained by analytically averaging the stochastic radiative transfer equation, are also used for estimating ensemble-averaged radiative properties. Contrary to the numerical averaging method, another Monte Carlo technique was applied for solving these equations (see previous section). Since only the bulk cloud statistics (see previous section) are used in the radiative calculations, the mean radiative properties obtained by this method are considered as approximations of the true radiative properties. Note, that using the approximated equations allows one to significantly speed-up (more that 1-2 factors of ten) calculations of the ensemble-averaged radiative properties. We will use subscript "app" to denote the approximated radiative properties.

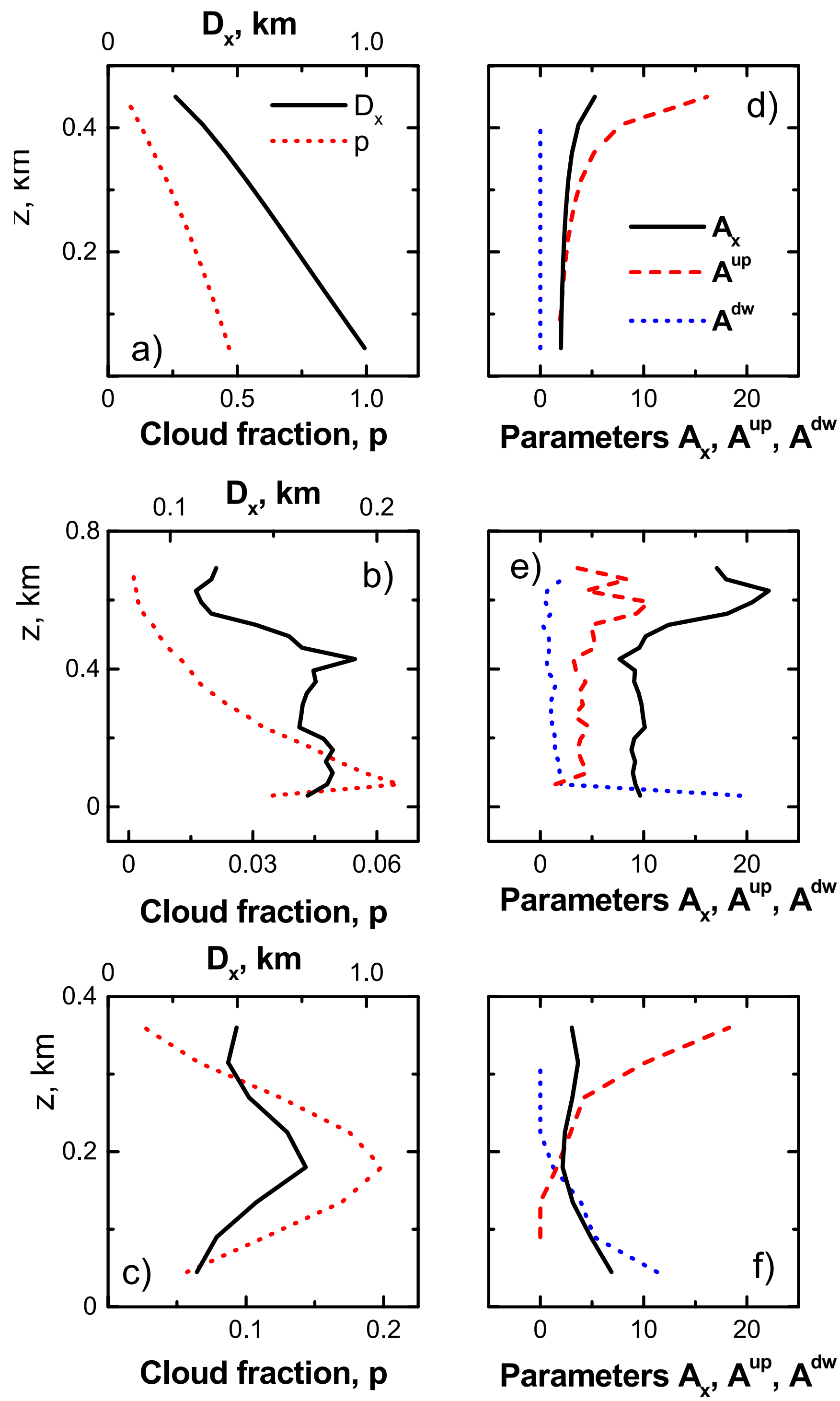

Figure 2. Left column: The vertical profiles of the cloud fraction, p, and the mean cloud horizontal (x-direction) chord, D_x. Right column: Vertical distribution of parameters A_x (x-direction), D_x (zenith-direction) and A^{dw} (nadir-direction). These bulk cloud statistics were obtained for the Boolean Figures a) and d), LES Figures b) and e), and MISR Figures c) and f) cloud fields.

To evaluate the accuracy of the approximated Eqs. (3) and (4), we use the corresponding relative differences (errors)

$$\delta F = \frac{\left(F_{ref} - F_{app}\right)}{F_{ref}} \times 100\,,\ \% \qquad (7)$$

where symbol F denotes radiative properties (e.g., mean flux, mean absorption).

Both the geometrical and optical properties of clouds must be known to calculate the radiative properties. To introduce significant vertical variability of cloud optical properties, we used different artificial vertical profiles of the extinction coefficient $\sigma(z) = \sigma_k$ and the single scattering albedo $\omega_0(z) = \omega_{0,k}$ with strong vertical gradients. Two types of vertical profiles were applied. The first type included vertical profiles of the extinction coefficient $\sigma(z)$ and the single scattering co-albedo, $1 - \omega_0(z)$, which increased with altitude inside a cloud layer (from cloud base to cloud top). Inversely, the second type included vertical profiles of $\sigma(z)$ and $1 - \omega_0(z)$, which decreased with altitude (from cloud top to cloud base). We found that the accuracy of approximated equations is almost independent of the vertical profile types, therefore we present results that correspond to the first type (Figure 3) only. For the LES field we used values of the extinction coefficient provided by the LES model, but the vertical variability of the single scattering albedo was introduced similar to the Boolean and MISR fields (Figure 3). To describe scattering by cloud droplets, we applied scattering phase function C1

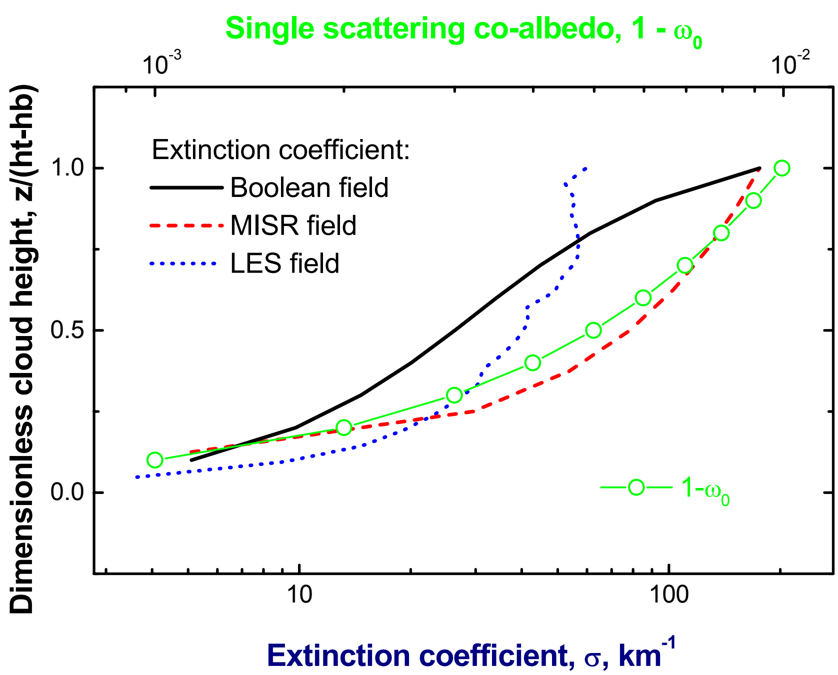

Figure 3. The vertical profiles of the extinction coefficient and single scattering co-albedo used for radiative calculations. Since the considered cloud fields had different geometrical thickness ($\Delta h = ht - hb$), these vertical profiles were shown as functions of dimensionless cloud height.

(Deirmendjian 1971). The latter was assumed to be constant and the same for all considered cloud fields. For obtaining the reference radiative properties and the approximated ones, we used the same optical characteristics.

While the radiative calculations were performed for a set of SZA, below we demonstrate results for two extreme values: SZA=0 and SZA=70. Since the accuracy of approximated equations may be different for direct (unscattered) and diffuse radiation, we present corresponding comparisons for both the mean direct radiance, vertical (upward and downward) fluxes and the mean cloud absorption. Here we consider only the absorption of the water droplets. The latter is proportional to the mean order of photon scattering in the kth layer.

Figure 4 shows the vertical profiles of the mean radiative properties. As can be seen for the majority of cases, the relative differences between the exact (the numerical averaging method) and approximated (the analytical averaging method) radiative properties do not exceed 10%. Note that these good agreements were obtained for ensemble-averaged radiative properties and artificial Markovian cloud fields. Therefore, the question arises: How well will the suggested statistical approach perform for a single realization of real cloud fields? In other words, is the suggested approach robust?

LES and MISR Cloud Fields

Equations (3) and (4) for the mean radiance were obtained by averaging over a set of cloud realizations (ensemble-averaged statistics). his makes it necessary to study a large group of such realizations, which as a rule the researcher does not have. To experimentally determine these statistical properties, one can apply the generally used ergotic assumption. In this case, the sought characteristics can be obtained from a single, sufficiently large realization (domain-averaged statistics). In other words, it is assumed that the domain-averaged statistics and the ensemble-averaged statistics are interchangeable. Note, the ergotic assumption is valid for Markovian random fields.

In this section, the validation analysis is similar to that described earlier, except that only domain-averaged statistics were taken into account. The bulk cloud statistics (Figure 2) were obtained from the single cloud realizations of the MISR and LES cloud fields (Figure 1). It is worth noting that (1) the LES cloud field contains a lot of small clouds (cloud horizontal size D≤1km), and (2) the MISR is composed of a few large clouds (D ~5km) surrounded by small clouds. Similar to the Boolean cloud field, the domain-averaged radiative properties were calculated by two independent methods. The full 3D cloud geometry was used in the first method (reference), and only the bulk cloud statistics (Figure 4) were applied for the second method (approximation).

Results of the radiative calculations are shown in Figures 5 and 6. For the LES cloud field, there is reasonable agreement between the exact and approximated results (Figure 5): while the maximum relative differences are about 25%, for the majority of cases the accuracy of approximated radiative calculations does not exceed 10%. The similar results were obtained for the MISR cloud field (Figure 6), except the maximum relative differences can be as great as 40%. Note, the differences between the exact and approximated domain-averaging results can be determined by two factors. First, the Markovian approximation cannot be appropriate for a given 3D cloud realization (cloud sample). Second, the sample size can not be sufficiently large for obtaining the cloud statistics.

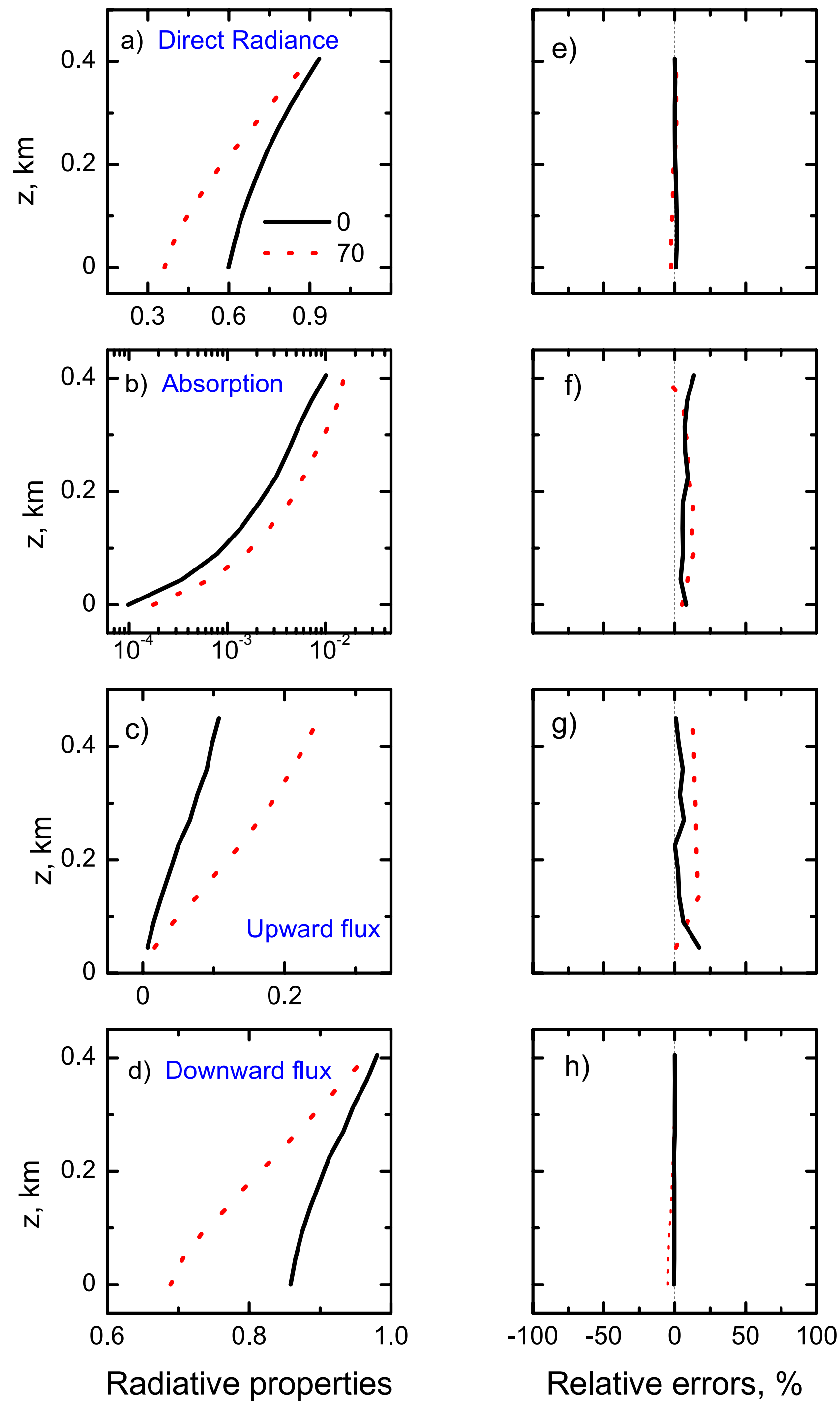

Figure 4. The ensemble-averaged radiative properties corresponding to the Boolean cloud fields. Left column: The mean vertical profiles of the direct radiance a), cloud absorption b), upward flux c) and downward flux d) that were obtained by using the numerical averaging method (reference). Right column: The relative differences between the mean vertical profiles that were calculated by using the numerical averaging method and the analytical averaging method (approximation).

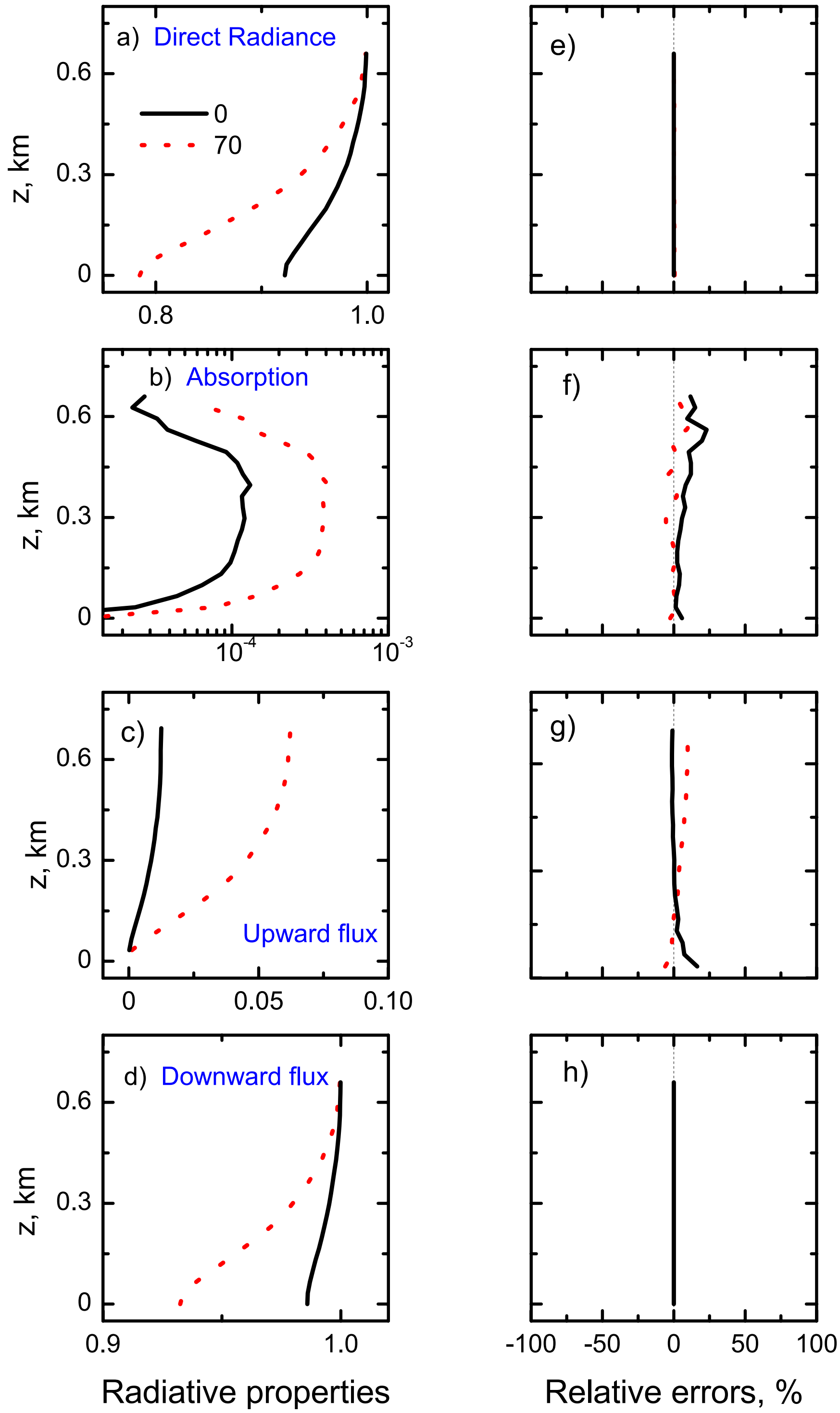

Figure 5. The domain-averaged radiative properties corresponding to the LES cloud fields. Left column: The mean vertical profiles of the direct radiance a), cloud absorption b), upward flux c) and downward flux d) that were obtained by using full 3D cloud structure (reference). Right column: The relative differences between the mean vertical profiles that were calculated by using full 3D cloud structure and the bulk cloud statistics (approximation).

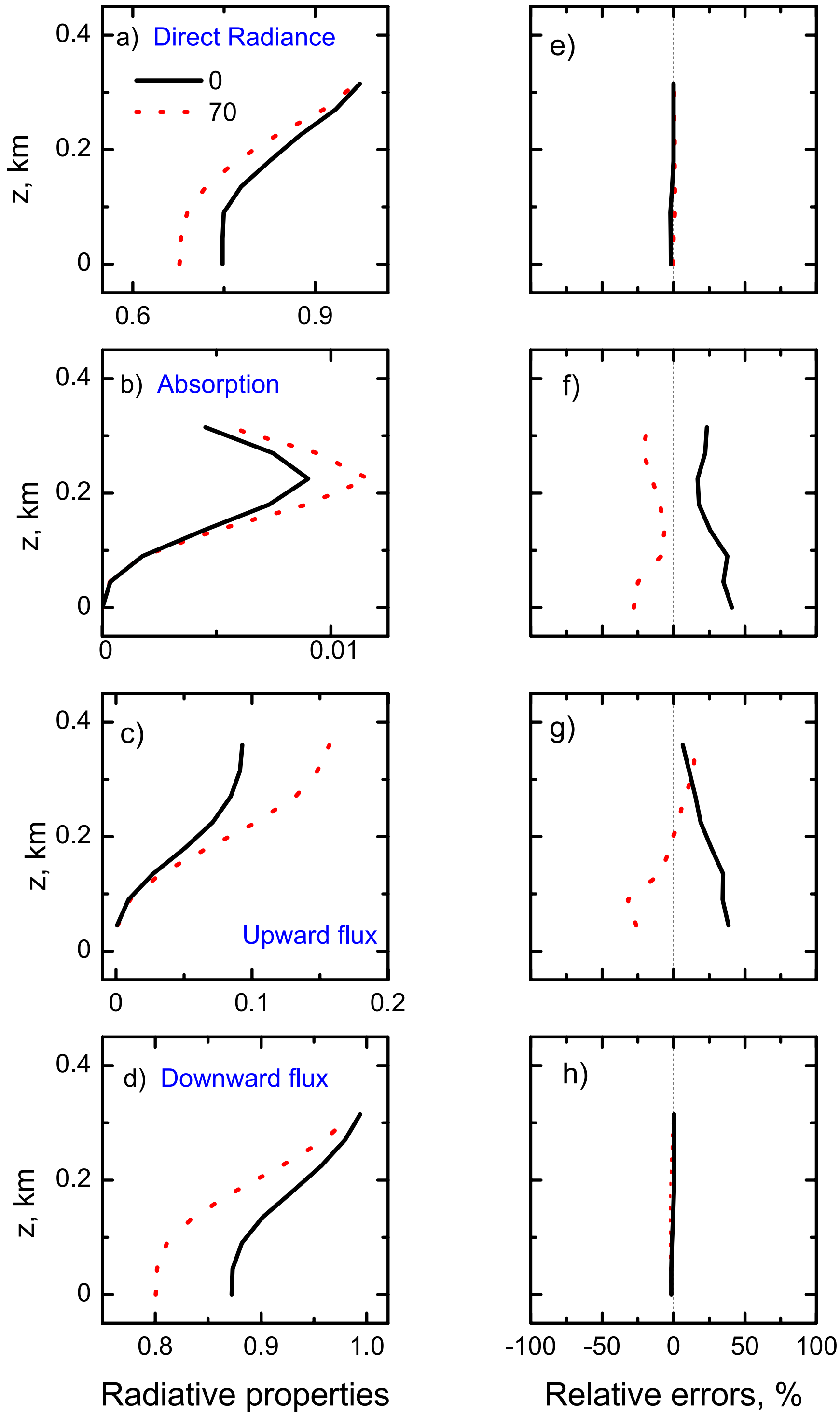

Figure 6. The same as in Figure 5, except that these results correspond to the MISR cloud field.

Obviously, the sample size of a given realization is a function of a cloud type (e.g., mean horizontal size D, and the nadir-view cloud fraction N_{nadir}). Here, we can only say that, for small (D ~1km) marine cumulus clouds , it should be ~10×10 km^2 and larger. In other words, a given cloud sample should contain clouds in abundance.

Conclusion

The validation analysis of the suggested statistical approach was performed by using 3D broken cloud fields produced by (1) Boolean stochastic model, (2) LES models and (3) MISR cloud retrieval. The LES and MISR cloud fields represent a marine boundary layer broken clouds at the ARM TWP site. It was demonstrated that the suggested approach allows one to estimate the ensemble-averaged absorption and vertical radiative fluxes with reasonable accuracy (~10%). The robustness of these equations was estimated by comparing the domain-averaged radiative properties obtained by using (1) the full 3D cloud structure of a given cloud sample (reference) and (2) the bulk cloud statistics (approximation). Single realizations of the LES (domain size 10×10 km^2) and MISR (domain size ~30×30 km^2) cloud fields were applied for this robust analysis. We found the suggested approach allows one to estimate the domain-averaged albedo and transmittance with similar accuracy (~15%). The errors of the approximated radiative calculations associated with different cloud types and the sample size require further investigation.

Acknowledgments

This work was supported by the Office of Biological and Environmental Research of the U.S. Department of Energy as part of the ARM Program and by NASA under contract number 121164 with NASA/Jet Propulsion Laboratory (JPL).

Corresponding Author

E. I. Kassianov, Evgueni.Kassianov@pnl.gov, (509) 372-6535

References

Benner, T. C., and J. A. Curry, 1998: Characteristics of small tropical cumulus clouds and their impact on the environment. *J. Geophys. Res.*, **103**, 28,753-28,767.

Clothiaux, E. E., K. P. Moran, B. E. Martner, T. P. Ackerman, G. G. Mace, T. Uttal, J. H. Mather, K. B. Widener, M. A. Miller, and D. J. Rodriguez, 1999: The Atmospheric Radiation Measurement cloud radars: Operational modes. *J. Atmos. Sci.,* **56**, 819-827.

Deirmendjian, D., 1971: *Electromagnetic Radiation Scattering by Spherical Polydisperse Particles*. Mir, p. 298

Kassianov, E. I., 2000: A generalization of stochastic radiative transfer model: Multiple broken layers. In *Proceedings of the Tenth Atmospheric Radiation Measurement (ARM) Science Team Meeting,* U.S. Department of Energy, Washington, D.C. Available URL: http://www.arm.gov/docs/documents/technical/conf_0003/kassianov-ei.pdf

Kassianov, E. I., T. P. Ackerman, R. T. Marchand, and M. Ovtchinnikov, 2001: Multi-angle satellite retrieval of cumulus thickness at the ARM TWP site: Validation tests. This Proceedings.

Marchuk, G., G. Mikhailov, M. Nazaraliev, R. Darbinjan, B. Kargin, and B. Elepov, 1980: *The Monte Carlo Methods in Atmospheric Optics.* Springer-Verlag, New York.

Stoyan, D., W. S. Kendall, and J. Mecke, 1995: *Stochastic Geometry and its Applications*. J. Wiley & Sons, Chichester.