A New Engine for Radiation Parameterization: Adjoint Perturbation and Selection Rule Methods for Broadband Fluxes

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Introduction

The purpose of this portion of the research is to develop new, rapid means for computing broadband and spectral fluxes and heating rates in both the shortwave and infrared (IR). These methods lead to nearly an order of magnitude increase in computational speed. The approach is a hybrid between adjoint perturbation and selection rule methods. Final tests of these methods are being performed. The algorithms are being incorporated into the Colorado State University (CSU) general circulation model (GCM).

Adjoint Perturbation Method

Among the implementation issues being addressed are the following:

- stability of the solution
- optimum selection of base states
- efficient implementation of the algorithms
- generating useful documentation.

An overview of the algorithm is required to understand the practical requirements. There are three stages that lead to the development of the adjoint perturbation method. The first is the calculation of the fluxes:

$$\frac{dF}{dz} - AF = \Gamma(z)$$

The generalized solution for the ith layer is

$$\begin{aligned} F_i(z) &= c_i^+ T_i^1(z) + c_i^- R_i^1(z) + S_i^+(z_i) T_i^1(z) \\ &\quad + S_i^-(z_{i+1}) R_i^1(z) - S_i^+(z) \\ F_i(z) &= c_i^+ R_i^2(z) + c_i^- T_i^2(z) + S_i^+(z_i) R_i^2(z) \\ &\quad + S_i^-(z_{i+1}) T_i^2(z) - S_i^-(z) \end{aligned}$$

Comment: the entire formulation is in physical space coordinates (z), not optical thickness (τ). The computational complexity is $O(N_l)$, requiring $2N_l+2$ coefficients, determined by a tri-diagonal solver.

Continuous Analytic Solution of the Adjoint Operator

The defining equation is

$$\frac{dF_a}{dz} - A^T F_a = -\delta(z - z_o)\,\mathbf{h}$$

The adjoint boundary conditions are

$$\begin{aligned} &F_a^+(z_{n+1})\,F^+(z_{n+1}) - F_a^+(z_1)\,F^+(z_1) = 0 \quad \text{and} \\ &F_a^-(z_{n+1})\,F^-(z_{n+1}) - F_a^-(z_1)\,F^-(z_1) = 0. \end{aligned}$$

The solution is

$$\begin{aligned} F_{a_i}^+(z, z_o) &= a_i^1 T_{a_1}^i(z) + a_i^2 R_{a_1}^i(z) + S_i^+(z_i)\, T_i^1(z) \\ &\quad - \left(e_{22}^i(z - z_o)h^+ + e_{12}^i(z - z_o)h^-\right) H(z - z_o) \\ F_{a_i}^-(z, z_o) &= a_i^1 R_{a_2}^i(z) + a_i^2 T_{a_2}^i(z) + S_i^+(z_1)\, T_i^1(z) \\ &\quad - \left(e_{21}^i(z - z_o)h^+ + e_{11}^i(z - z_o)h^-\right) H(z - z_o) \end{aligned}$$

Comment: The adjoint operators contain unbounded terms in T_a, R_a, and in the e_{ij}. These adjoint functions have no simple physical interpretation, but are derivable from the physically meaningful R and T by a transformation. As with the standard fluxes, the adjoint coefficients are obtained by a tri-diagonal solver. However, since $2N_l$ coefficients must be obtained for every layer and there are N_l layers, the complexity increases to $2N_1^2$. A data base of these coefficients must be created for the base state. Note that the boundary conditions for the adjoint operators are prescribed by the physical fluxes exiting the medium. Thus, if the emerging fluxes approach zero, as they would in a strongly absorbing medium, the adjoint will diverge. Clearly, because of the dependence of the adjoint boundary conditions on the physical fluxes, the adjoint cannot be used to calculate fluxes. As such, the adjoint's utility lies in the calculation of the perturbations, or speaks to the possibility of solving the internal fluxes of a plane-parallel medium if the fluxes exiting the medium are known.

The continuous flux solution for the perturbation about the base state is obtained from

$$F_{new}(z_o) = F_{basestate}(z_o) - \sum_{i=1}^{N_1} \left(\alpha_i(z_o)\delta\gamma_i^1 + \beta_i(z_o)\delta\gamma_i^2\right)$$

Comment: Stability is clearly dependent on that of the adjoint coefficients. The $\delta\gamma$ are the difference between the final and the base state, while the other two coefficients, α and β, are properties of the base state and are dependent on the vertical coordinate.

Selection Rule Method

The selection rule method was developed to work in tandem with the adjoint perturbation. Its purpose is to bypass the instability problem that occurs when the absorption is large. This is accomplished by examining the single scattering albedo and scattering coefficient of each layer in the atmosphere. If all of the layers are strongly absorbing or weakly scattering (the thresholds are defined by the user), then in the shortwave, only the direct beam contributes, without the need for a full up two-stream calculation. If any of the layers violate this criterion, then a complete radiative transfer solution must be performed. In the clear-sky case, only 5 out of 52 radiative transfer calculations are required for the broadband fluxes and heating rates as determined by the K distribution method. For the cloudy atmosphere, the number of full up calculations varies according to where the cloud is situated.

Cirrus clouds are the most demanding, requiring 12 out of 52 full calculations. The reason is that these clouds reside high in the atmosphere where absorption is weak on account of the low concentration of absorbing gases, while particle scattering is high. Whether or not the perturbation is applicable here depends on the scattering by other layers: if scattering prevails, the perturbation technique is applicable. The practical issues that must be addressed are 1) what is the optimal base state, and how is this selected? 2) how high can the threshold for the single scattering albedo be set for the perturbation method to be usable? Preliminary tests indicate that the perturbation/selection rule approaches can yield solution within 1% of the true solution and with a single scattering albedo as high as 0.975.