Radar remote sensing of rainfall

Doppler radar signal theory and spectral estimation

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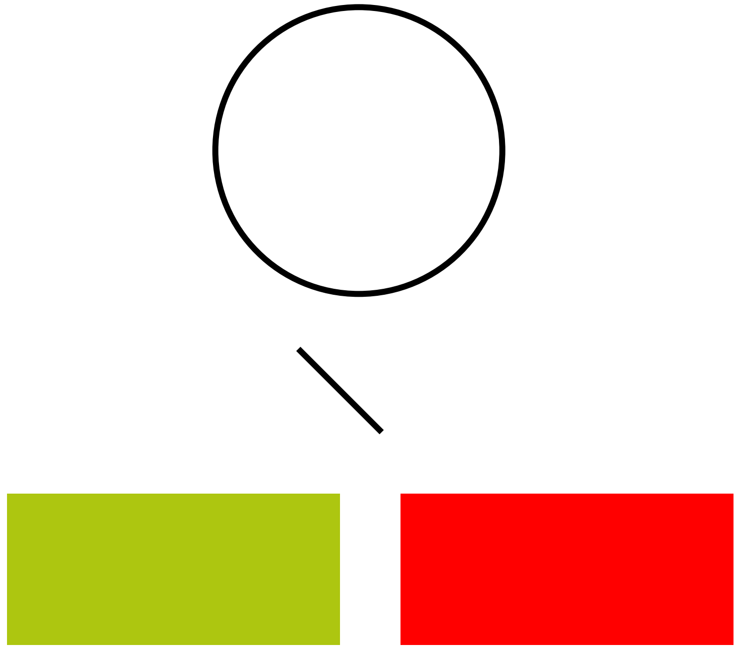
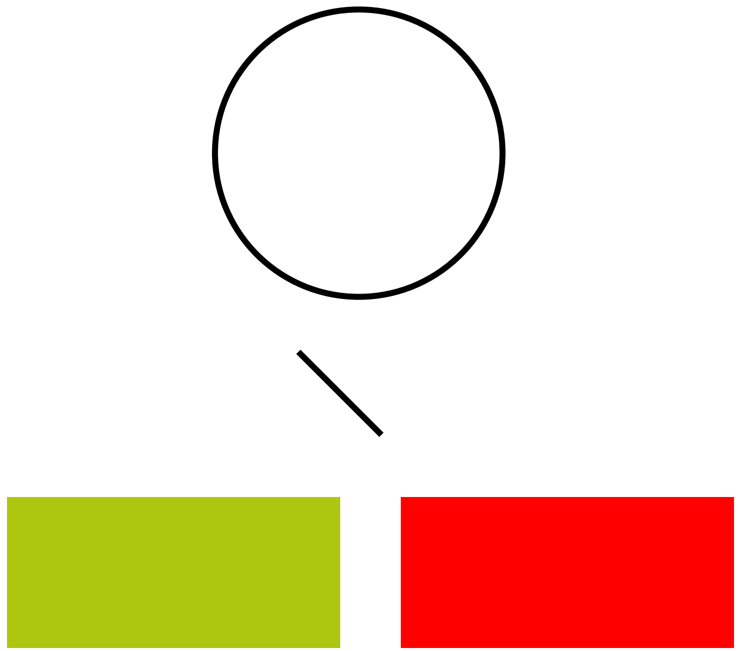
A bit about observations

How to describe an apple just by looking at it?



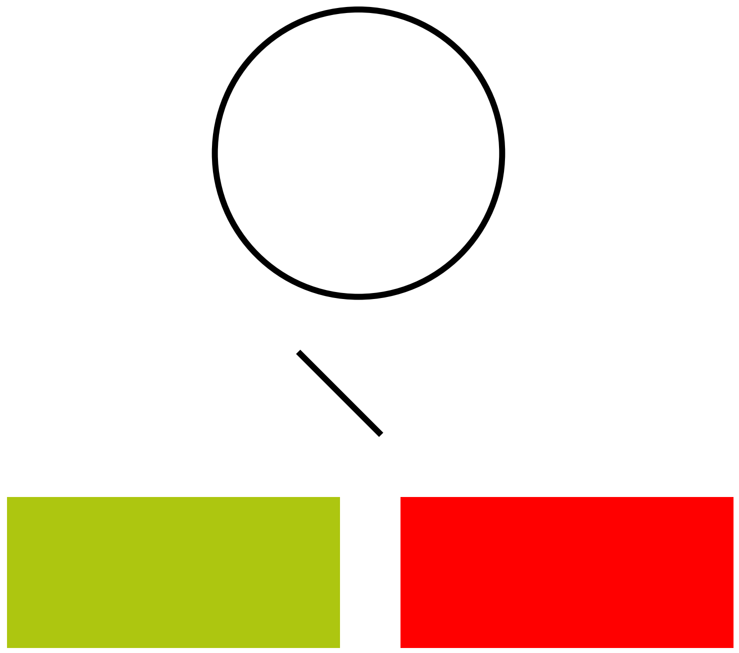
An apple is round and is green and/or red

And now backwards!



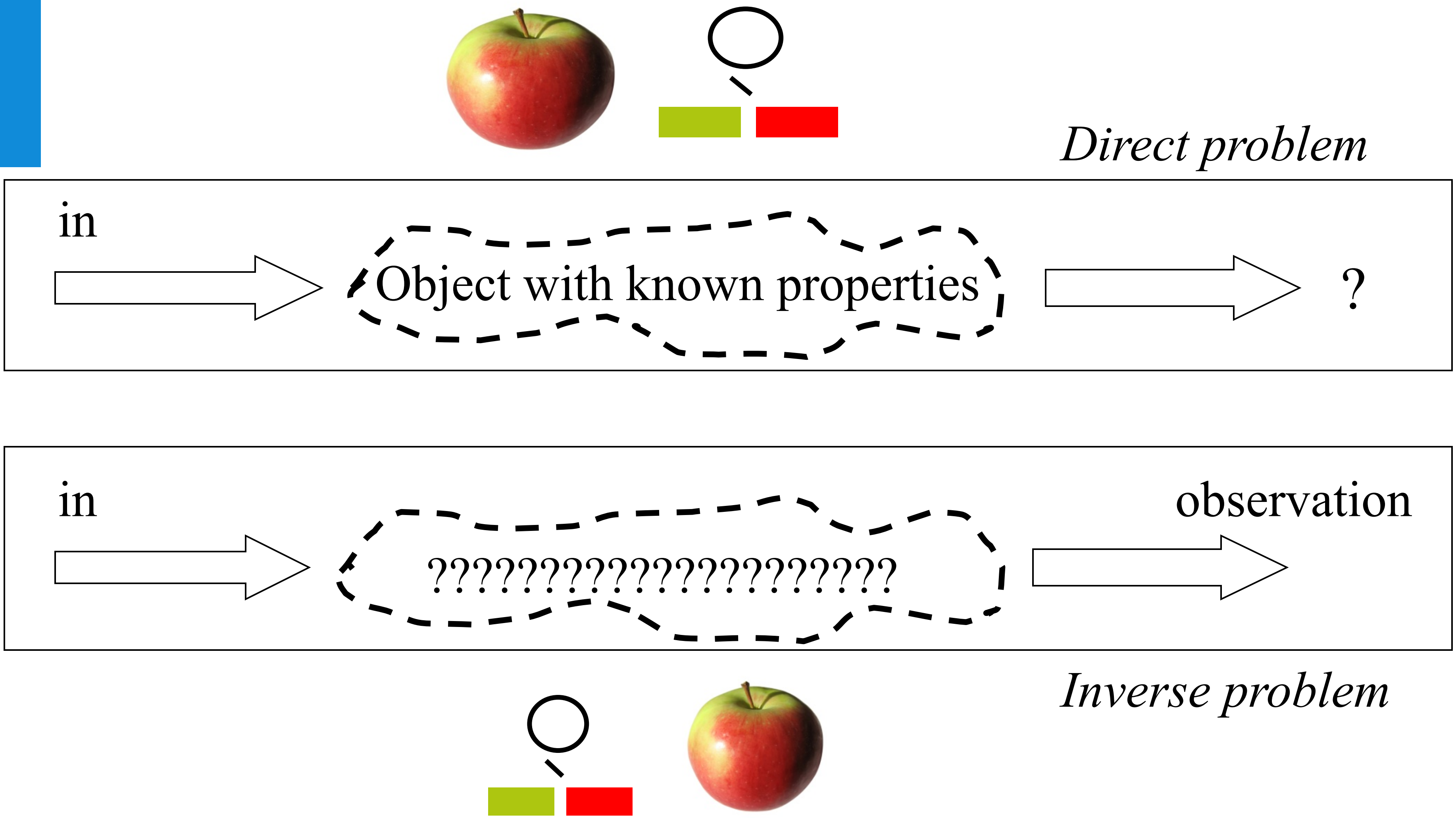
It is round, red and/or green, and therefore an apple?

We need a priori knowledge



When we know we are looking at apples, then we can we describe its properties:
it is a nice round, red and/or green apple.

In terms of observations



The complexity of the inverse problem

- How to describe the characteristics of an object
- With a limited number of observed parameters
- With sufficient accuracy?

What do we measure with a radar?

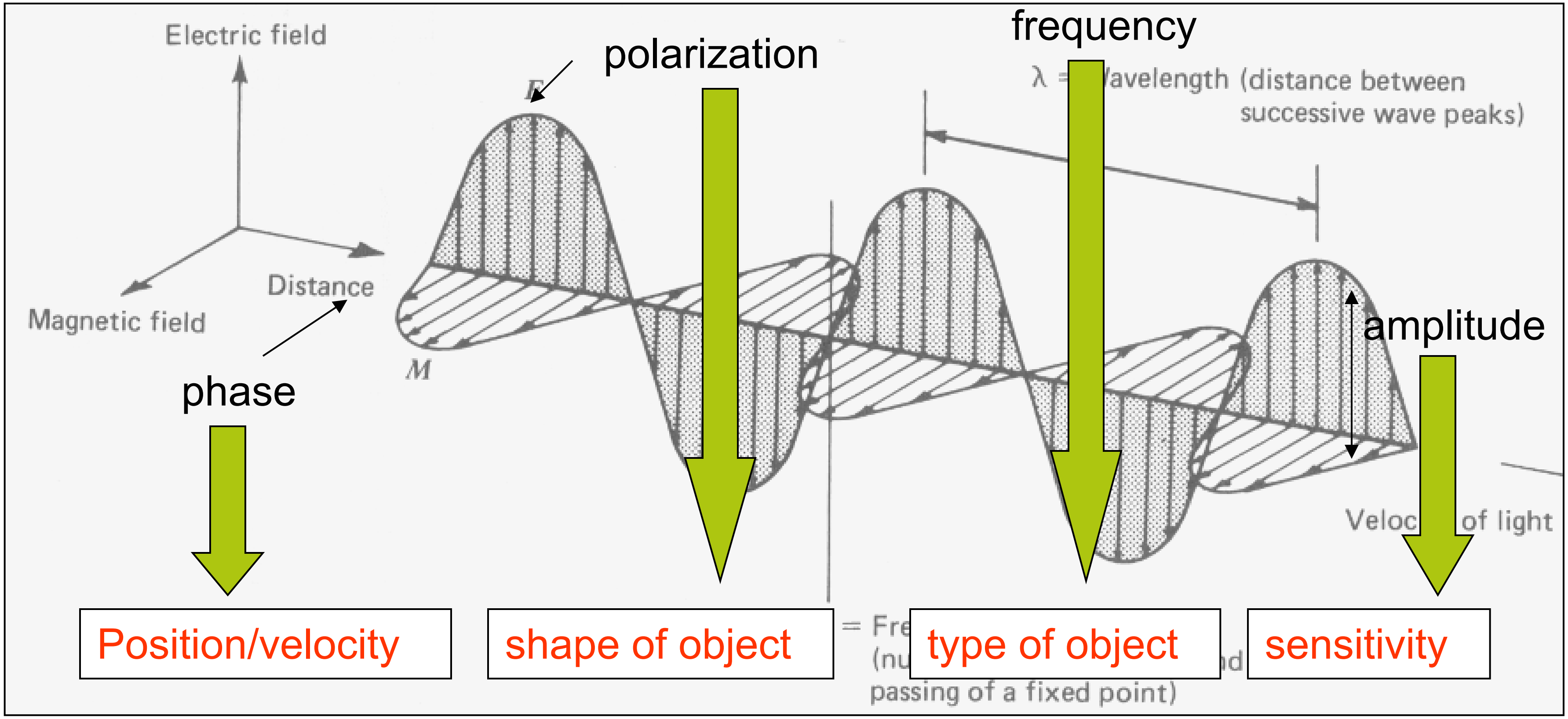
Signals

amplitude
phase
polarization
frequency

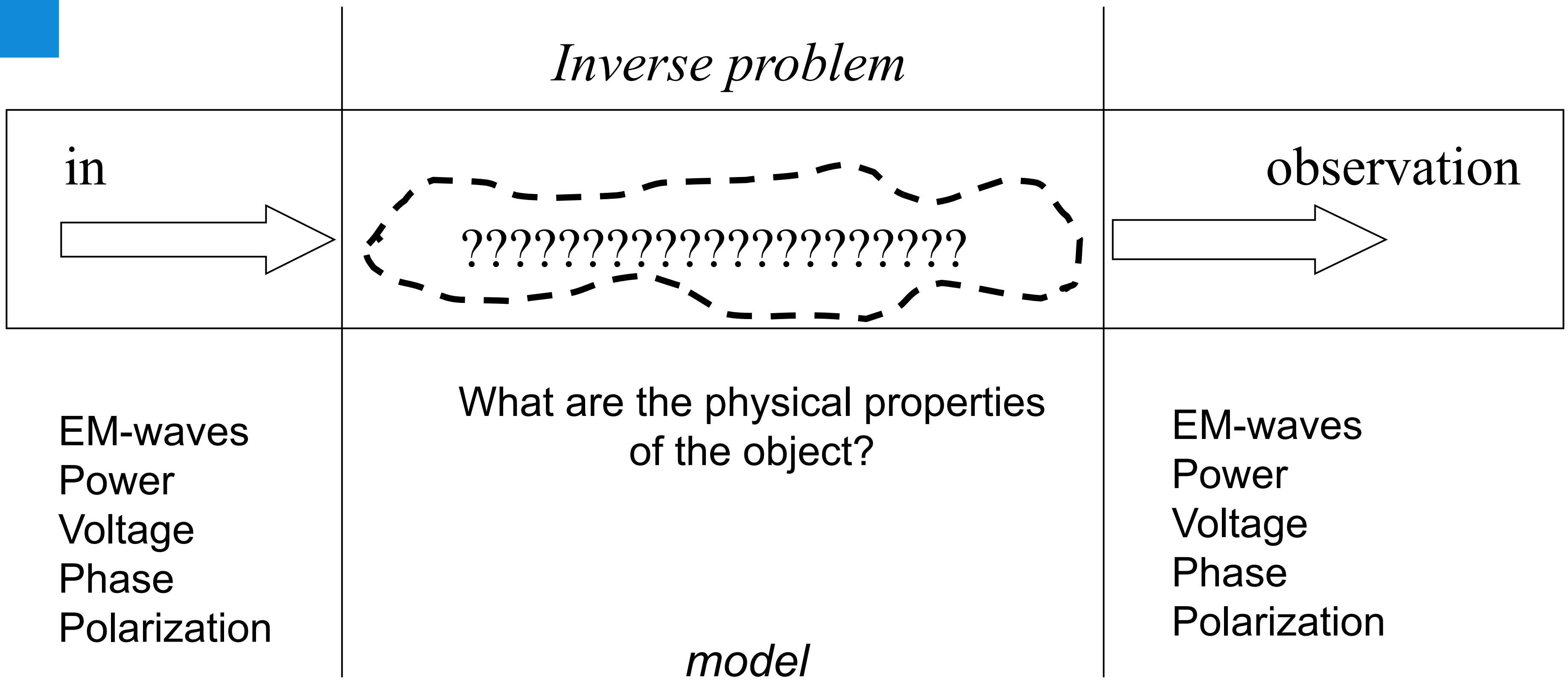
Without interpretation meaningless parameters!

For remote sensing: we want to derive the physical properties of system earth

How can we use EM-waves?



Signals and meaning



$$F\left(amplitude, phase, polarization, frequency\right)$$

Applications

$$F\left(amplitude, phase, polarization, frequency\right)$$

EM-waves
Power
Voltage
Phase
Polarization

→

Rainfall, clouds, wind, greenhouse gases
Sea temperature, soil moisture, vegetation
Ocean currents, polar ice, glaciers

↓

EM-waves
Power
Voltage
Phase
Polarization

→

Questions:
how much rain, ice, gas etc?

Signals and meaning, 2

$$out = F(x, y, z)$$

What is the accuracy of 'out'?

$$\left|\frac{\Delta out}{out}\right|^2 = \left|\frac{\Delta F(x,y,z)}{x}\Delta x\right|^2 + \left|\frac{\Delta F(x,y,z)}{y}\Delta y\right|^2 + \left|\frac{\Delta F(x,y,z)}{z}\Delta z\right|^2$$

So we must know the accuracy of x,y,z

And the required accuracy of the signal comes from the model. Suppose a simple model: the model output is related to the power P:

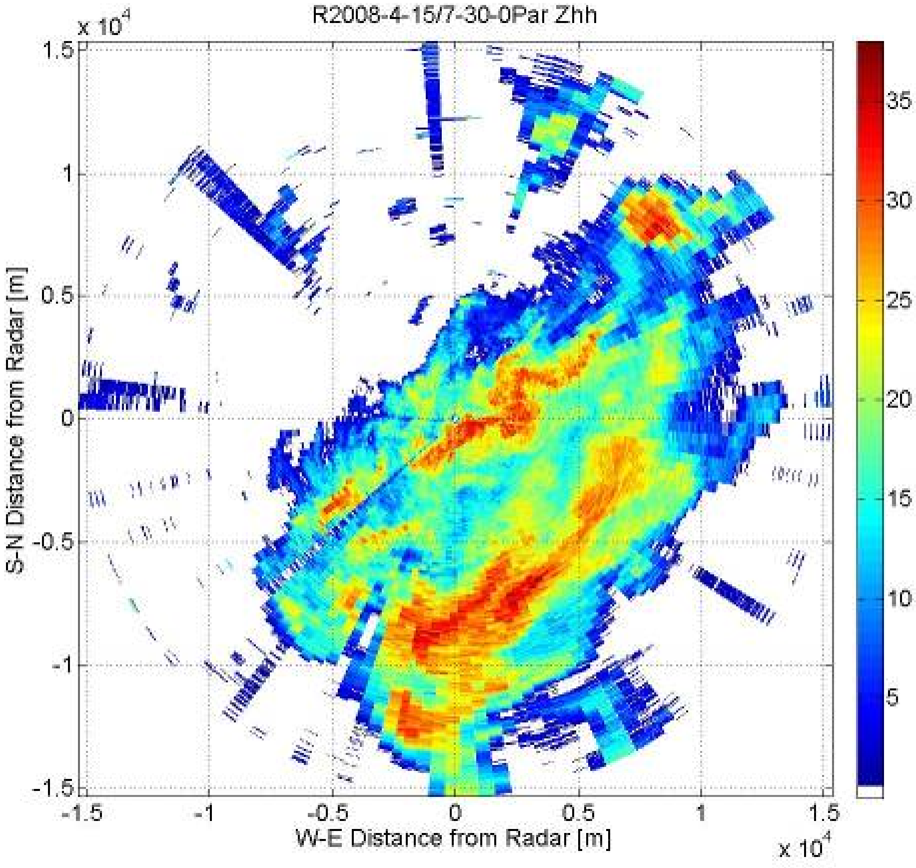
$$out = \alpha P^{\beta} \rightarrow \Delta out = \alpha\beta P^{\beta-1} \Delta P \rightarrow \frac{\Delta out}{out} = \frac{\alpha\beta P^{\beta-1} \Delta P}{\alpha P^{\beta}} = \beta \frac{\Delta P}{P}$$

$$\frac{\Delta P}{P} = \frac{1}{\beta} \frac{\Delta out}{out}$$

Needed accuracy of observed power

Required accuracy of parameter of interest

Example of model: weather radar



Radar reflectivity factor Z

$$Z = aR^b$$

Rainfall rate R

a,b constants

b~1.5:

If we want to know R

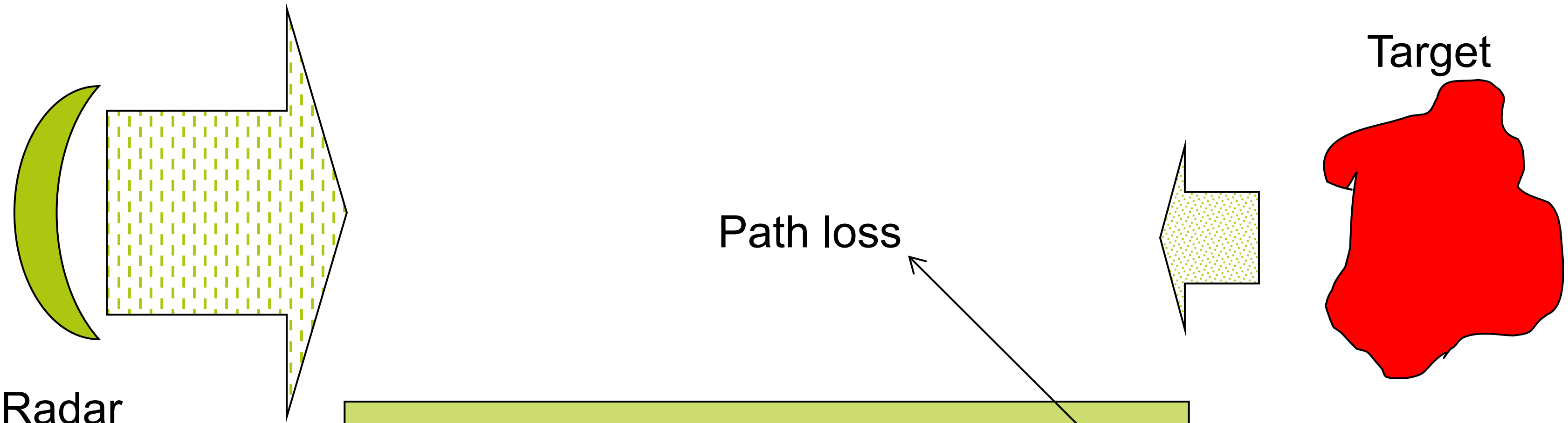
Within 10% then Z has to

be measured within 15%

How to achieve the required accuracy?

- Understand and quantify error sources
- Understand and quantify signal behaviour
- Reduce noise

Back to the radar equation



Path loss

$$P_{rec} = \frac{P_t G_t G_r \lambda^2 L_s \sigma}{(4\pi)^3 R^4} = \frac{C L_s \sigma}{R^4}$$

Radar cross section: target properties

System parameters

Example: the target is a volume filled with raindrops

Radar cross-section of volume

$$\sigma_{vol} = \sum_{k}^{N_{tot}} \sigma_k = \sum_{i}^{N_{m3}} \sigma_i \cdot Volume$$

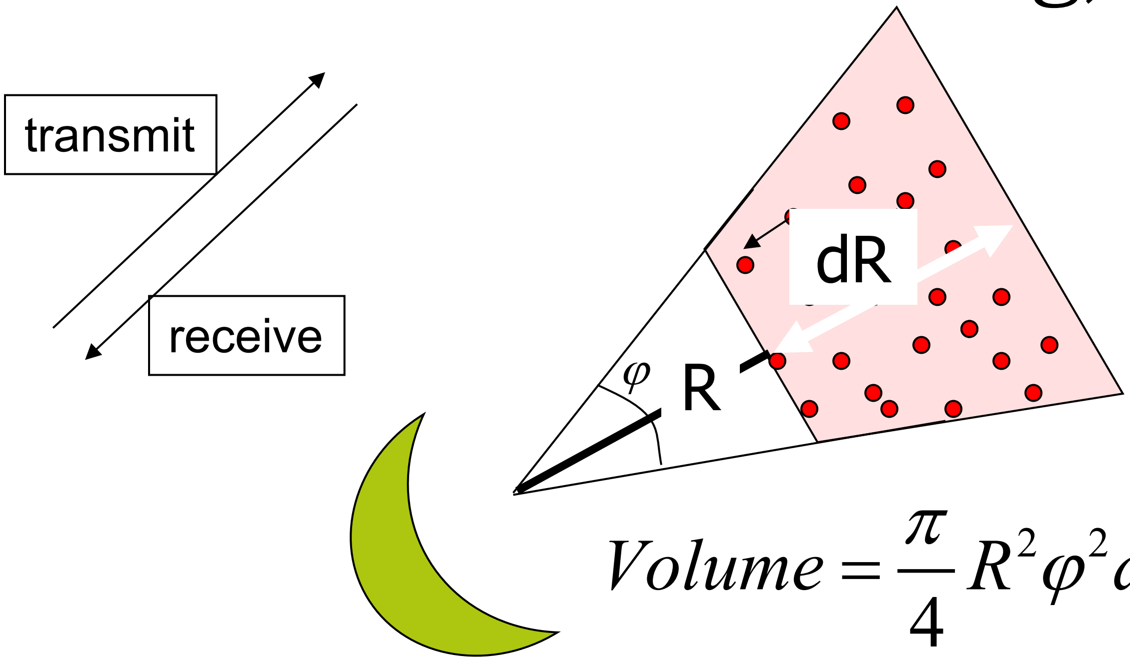
$$\sigma_{vol} = \eta \cdot Volume$$

reflectivity

Summation over complete volume

Summation over cubic meter

Radar equation for volume scattering, 1



$$Volume = \frac{\pi}{4} R^2 \varphi^2 dR$$

$$P_{rec} = \frac{P_t G_t G_r L_s \lambda^2}{(4\pi)^3 R^4} \sigma_{tot} = \frac{P_t G_t G_r L_s \lambda^2}{(4\pi)^3 R^4} \eta \cdot Volume = \frac{P_t G_t G_r L_s \lambda^2 \phi^2 dR}{64\pi^2 R^2} \eta$$

Radar equation for volume scattering,2

$$P_{rec} = \left(\frac{P_t G_t G_r \lambda^2 \pi \phi^2}{64\pi^2 R^2} dR \right) \cdot L_s \eta = C L_s \eta$$

Deterministic:
Calibration of system
To reduce errors

Stochastic:
Signal processing to reduce errors

Radar equation for volume scattering,3

Since the target is of stochastic nature, we need more measurements and integrate. The brackets denote time-average:

$$\langle P_{rec} \rangle = \left(\frac{P_t G_t G_r \lambda^2 \pi \phi^2}{64 \pi^2 R^2} dR \right) \cdot \langle L_s \eta \rangle = C \langle L_s \eta \rangle$$

How do we know how long we have to average?

We have to know the variance of the signal: theory of signal statistics

Signal statistics, time series, N samples

Voltage	$V[n], n = 1,2,....N$
Mean power	$\overline{P} = \frac{1}{N}\sum_{n} V[n]V^*[n]$
Variance	$\text{var}(\overline{P}) = \frac{\overline{P}^2}{N}\sum_{l=-(N-1)}^{(N-1)}(1-\frac{\lvert l\rvert}{N})\rho_P[l]$
Uncorrelated samples	$\rho_P[0] = 1 \wedge \rho_P[l] = 0 \;\; (l \neq 0) \rightarrow \text{var}(\overline{P}) = \frac{\overline{P}^2}{N}$

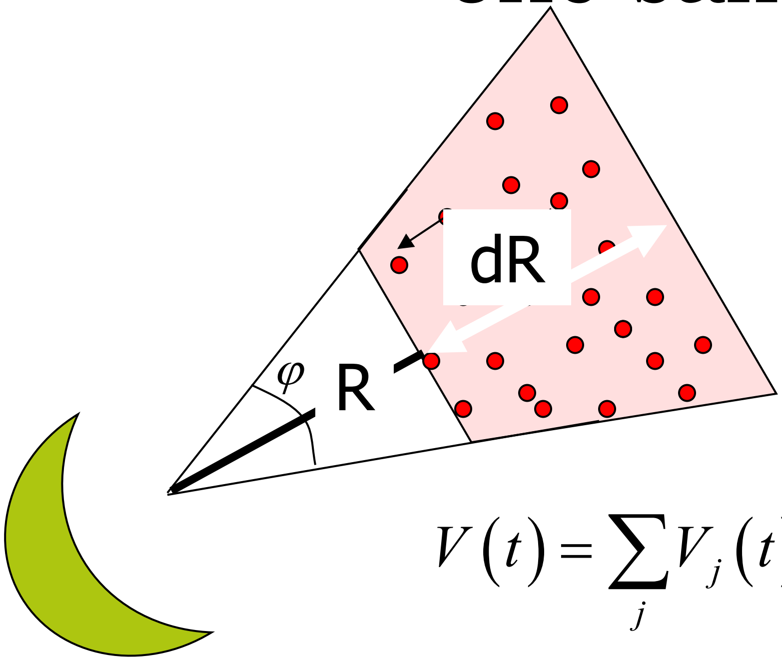
Estimation of mean power

The variance of the signal can be reduced with averaging

Q: How to determine the number of samples in advance?
A: We have to know the statistical properties of the signal.

Statistical distribution of power, voltage, phase

Signal statistics for volume scattering; one sample, many drops



Every drop *j* gives a complex voltage *Vj*
With amplitude and phase

$$V(t)=\sum_j V_j(t)=\sum_j \mathrm{Re}\left(V_j(t)\right)+\sum_j \mathrm{Im}\left(V_j(t)\right)$$

Phase of V(t) uniformly distributed
From statistics: many particles > central limit theorem:
Gaussian distribution of real and imaginary part

Statistical model of radar signal in case of volume scattering

$$V(t) = I(t) + jQ(t)$$

$$\langle I(t) \rangle = \langle Q(t) \rangle = 0$$

$$Var\left(I^2(t)\right) = Var\left(Q^2(t)\right) = \sigma^2$$

$$E\left(I(t_1)I(t_2)\right) = E\left(Q(t_1)Q(t_2)\right) = \sigma^2 \rho_0\left(t\right)$$

$$E\left(I(t_1)Q(t_2)\right) = E\left(Q(t_1)I(t_2)\right) = \sigma^2 \alpha_0\left(t\right)$$

Statistical model of radar signal: probability density functions

amplitude $$f(|V|) = \frac{|V|}{\sigma^2} \exp\left(\frac{-|V|^2}{2\sigma^2} \right)$$

phase $$f(\theta) = \frac{1}{2\pi}$$

power $$f(P) = \frac{1}{2\sigma^2} \exp\left(\frac{-P}{\overline{P}} \right) = \frac{1}{\overline{P}} \exp\left(\frac{-P}{\overline{P}} \right)$$

Probability density functions

Known pdf's:

Known mean, Variance etc

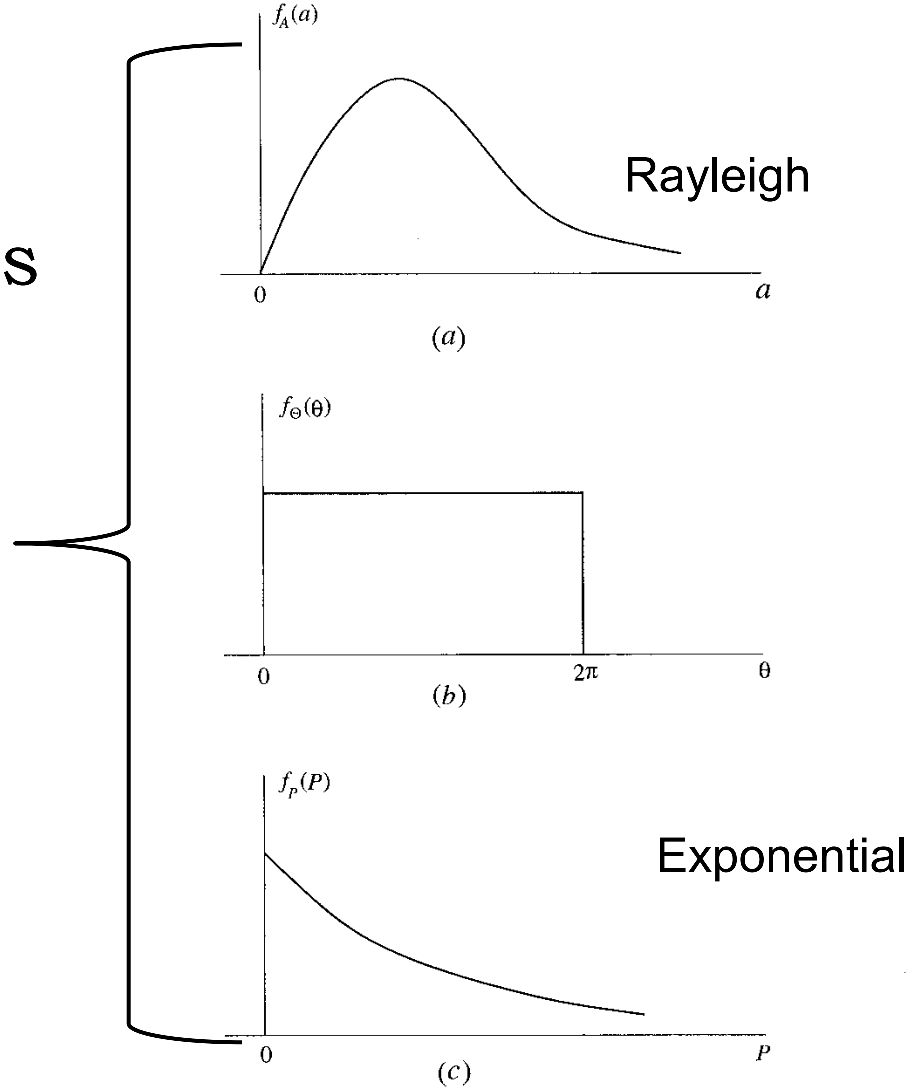


Fig. 5.33. Probability density function of (a) signal amplitude, (b) signal phase, and (c) signal power.

Statistical moments of voltage and power

$$\langle |V| \rangle = \sigma \sqrt{\frac{\pi}{2}}; \operatorname{var}\left(|V|\right) = \frac{4-\pi}{2}\sigma^2$$

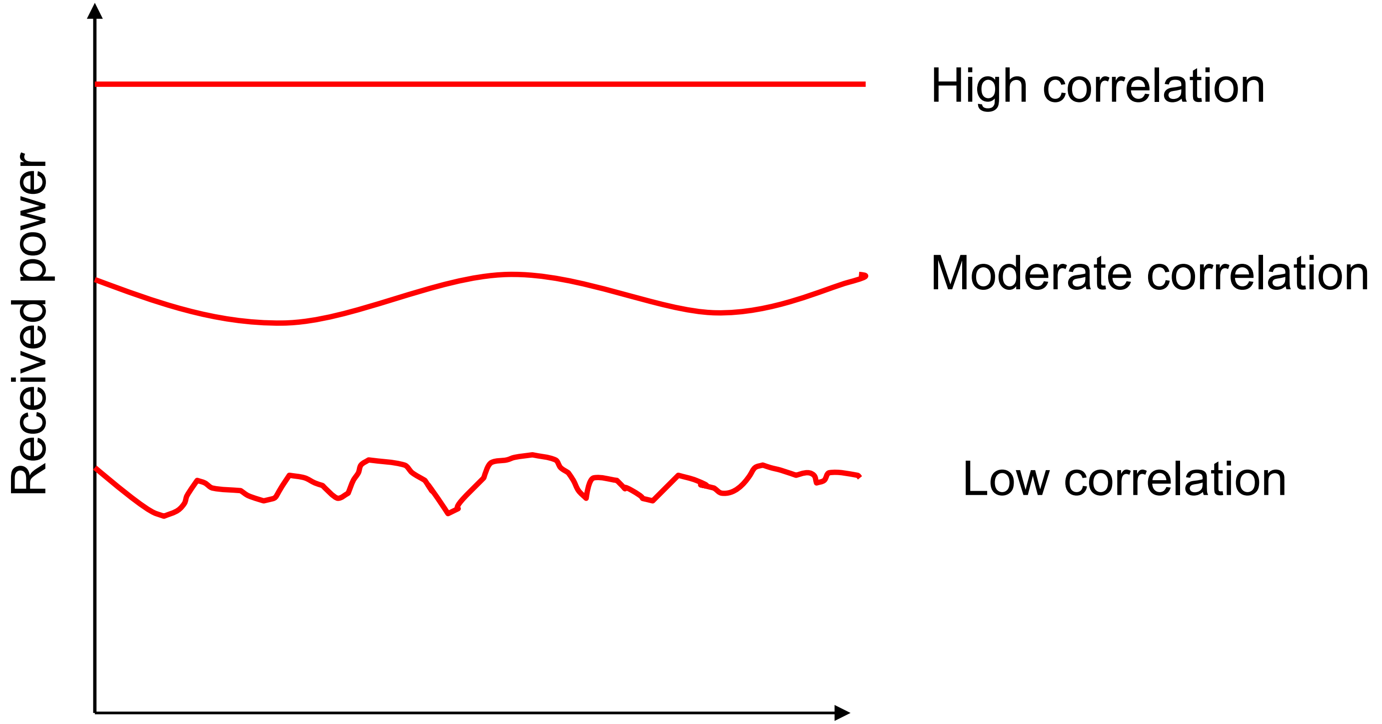
$$\langle P \rangle = \overline{P}; \operatorname{var}\left(P\right) = \overline{P}^2$$

One sample

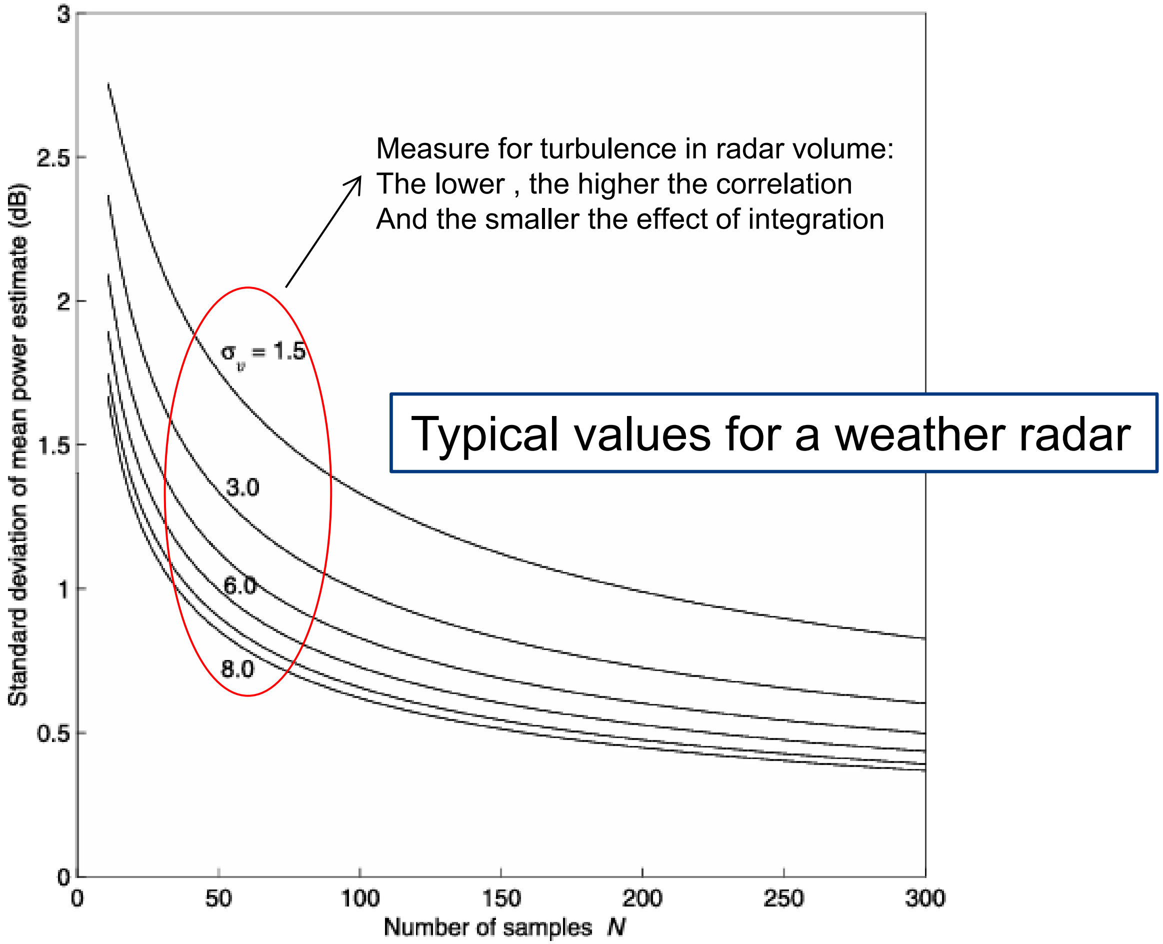
N samples

$$\rho_P[0] = 1 \wedge \rho_P[l] = 0 \;\; \left(l \neq 0\right) \rightarrow \operatorname{var}(\overline{P}) = \frac{\overline{P}^2}{N}$$

What is correlation?



Predicted accuracy based on signal model



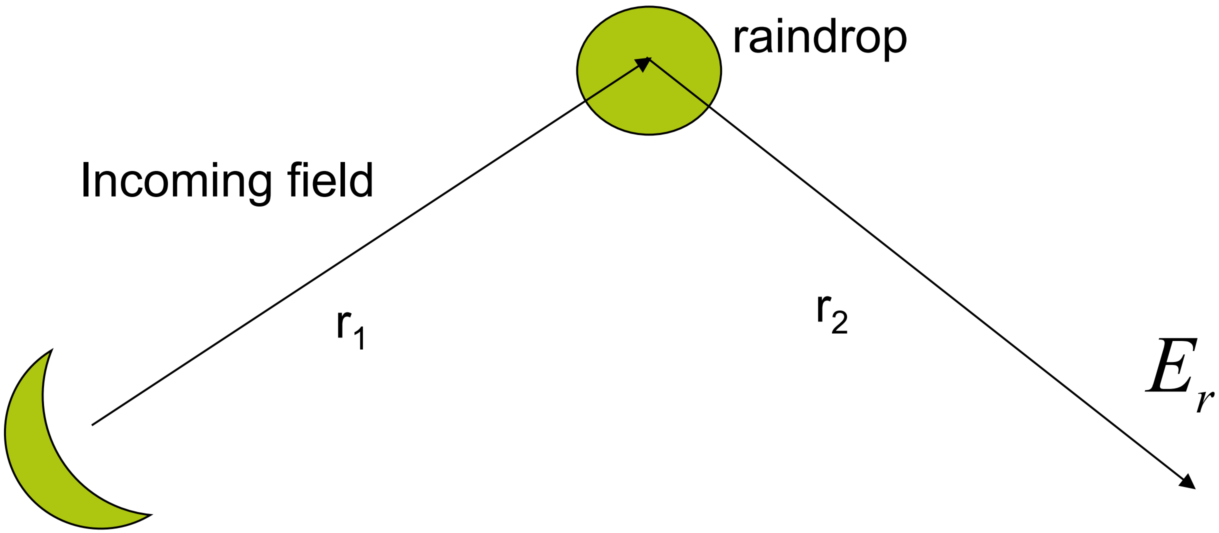
Trade-off integration time

Integration time ok

Integration time too long

The physical properties of the target may not change too much during the integration time: the result becomes meaningless!

Signal basics weather radar,1



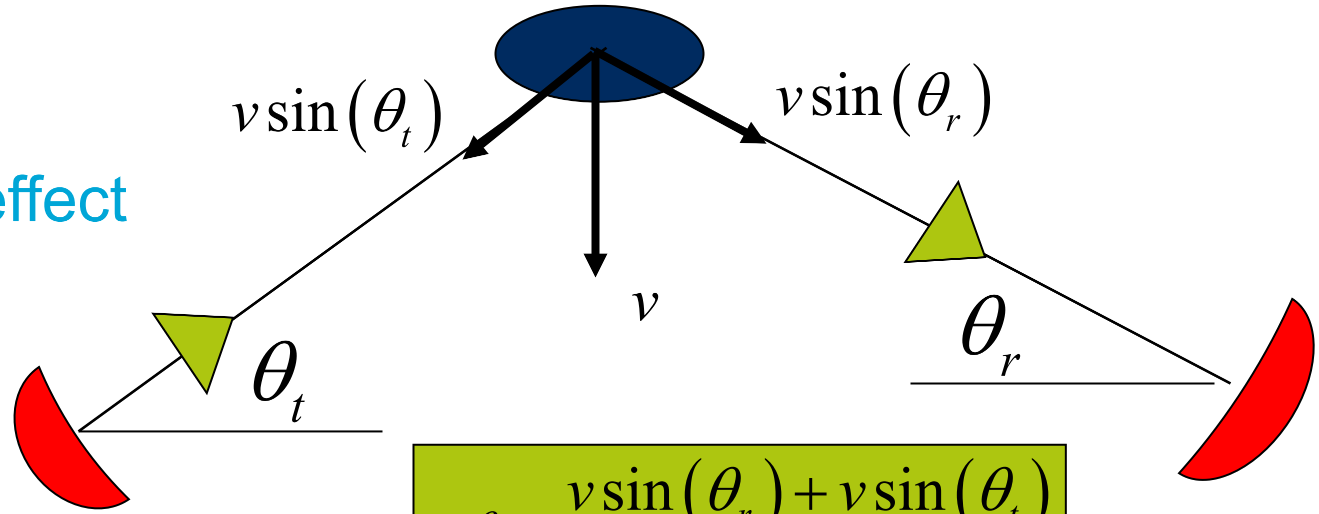
$$E_r \sim E_0 e^{j2\beta(r_1+r_2)}$$

$$\varphi = 2\beta\left(r_1 + r_2\right) \rightarrow \frac{\partial\varphi}{\partial t} = 2\beta\frac{\partial\left(r_1 + r_2\right)}{\partial t} = 2\beta\left(v_1 + v_2\right) = \varpi = 2\pi f$$

$$\beta = \frac{2\pi}{\lambda} \rightarrow f = \frac{2\left(v_1 + v_2\right)}{\lambda}$$

Signal basics weather radar,2

The Doppler effect



$$\Delta f = \frac{v\sin(\theta_r) + v\sin(\theta_t)}{\lambda}$$

Forward scatter:

$$\theta_r = \theta_t + \pi \rightarrow \Delta f = 0$$

Backscatter:

$$\theta_r = \pi - \theta_t \rightarrow \Delta f = \frac{2v}{\lambda}\sin(\theta_r)$$

Doppler shift is only representative for the velocity along the antenna beam

Signal basics weather radar,3

Maximum phase shift $\varphi_{\max} = \pm\pi$

$$\varphi_{\max} = 2\pi f T_0 = 2\pi \frac{2v}{\lambda} T_0$$

$$v_{\max} = \pm \frac{\lambda}{4T_0}$$

Maximum unambiguous Doppler velocity

Time between two samples

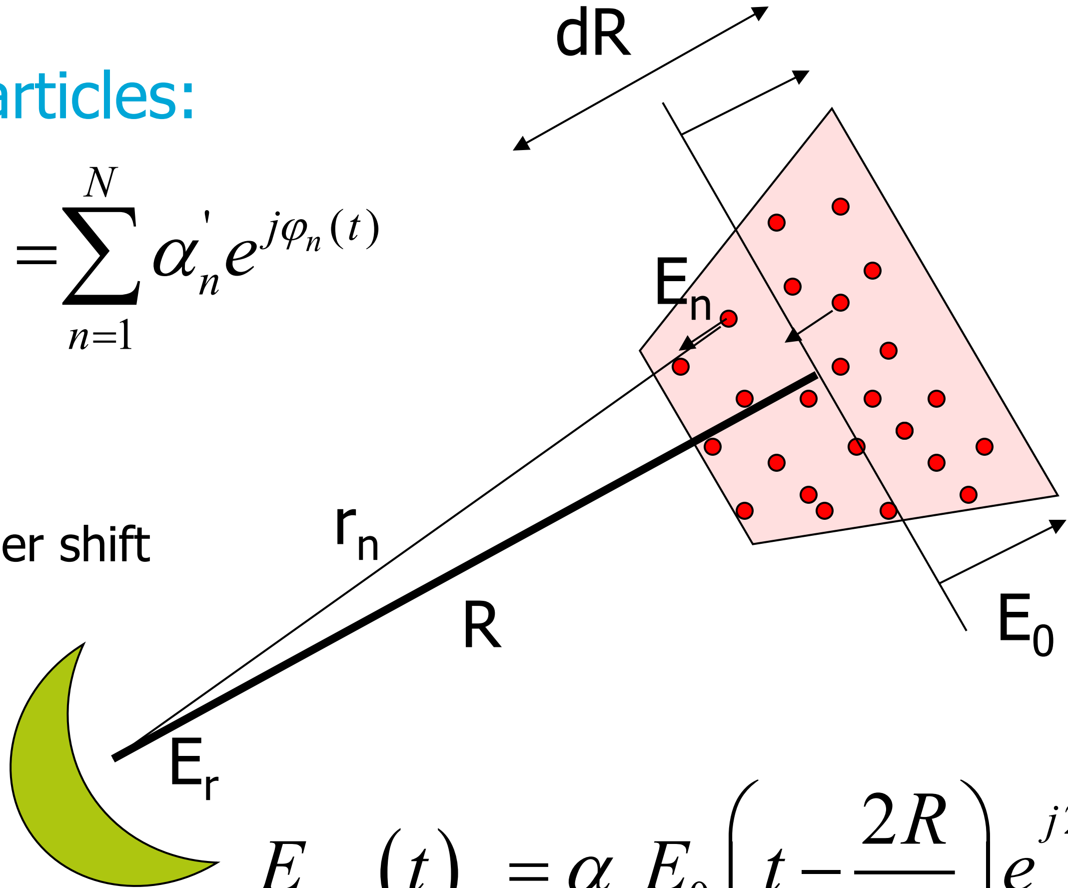
Signal basics weather radar,4

Scattering by N particles:

$$E_{tot}(t) = \sum_{n=1}^{N} E_n(t) = \sum_{n=1}^{N} \alpha_n' e^{j\varphi_n(t)}$$

$$\varphi_n(t) = \frac{2\pi r_n}{\lambda} + \frac{2v}{\lambda}t$$

Doppler shift



$$E_{r,n}(t)_r = \alpha_n E_0\left(t - \frac{2R}{c}\right) e^{j2\pi\frac{2v}{\lambda}t}$$

$$dR \ll R$$

Signal basics weather radar,5

$$E_{tot} = \sum_{n}^{N} E_{r,n}(t) = \sum_{n}^{N} \alpha_n E_0\left(t - \frac{2R}{c}\right) e^{j2\pi\frac{2v}{\lambda}t}$$

Received power

incoherent

$$P = \frac{1}{2}\left|E_{tot}\right|^2 = \frac{1}{2}\sum_{n}^{N}\left|\alpha_n\right|^2 +$$

coherent

$$\frac{1}{2}\mathrm{Re}\left\{\sum_{i}^{N}\sum_{j\neq i}^{N}\alpha_i\alpha_j E_{0,j}\left(t-\frac{2R}{c}\right)E_{0,i}^{*}\left(t-\frac{2R}{c}\right)e^{j2\pi\Delta f_{ij}t}\right\}$$

Signal basics weather radar,6

Doppler information is coded in the signal phase:

a power measurement is not sufficient

complex processing is required to retrieve the particle speed

e.g.fourier transforms, autocorrelation functions

Basis of spectral processing

one radar cell, one distance

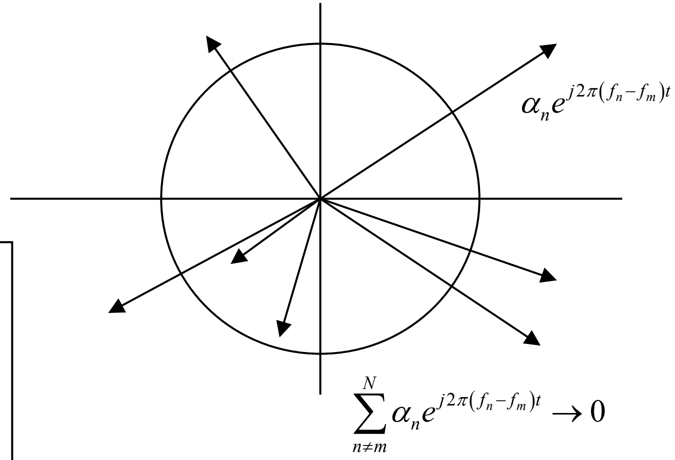
$$E_{tot} = \sum_{n}^{N} \alpha_n(t) e^{j2\pi \frac{2v_n}{\lambda} t} = \sum_{n}^{N} \alpha_n(t) e^{j2\pi f_n t}; \quad E_0\left(t - \frac{2R}{c}\right) = 1\, for\, sake\, of\, simplicity$$

$$E_{tot}(f_m) = \frac{1}{T}\int_{t}^{t+T}\left(\sum_{n}^{N} \alpha_n(t) e^{j2\pi f_n t} e^{-j2\pi f_m t}\right)dt = \frac{1}{T}\int_{t}^{t+T}\left(\sum_{n}^{N} \alpha_n(t) e^{j2\pi(f_n - f_m)t}\right)dt$$

$$E_{tot}(f_m) = \frac{1}{T}\int_{t}^{t+T}\left(\alpha_m(t) + \sum_{n\neq m}^{N} \alpha_n e^{j2\pi(f_n - f_m)t}\right)dt$$

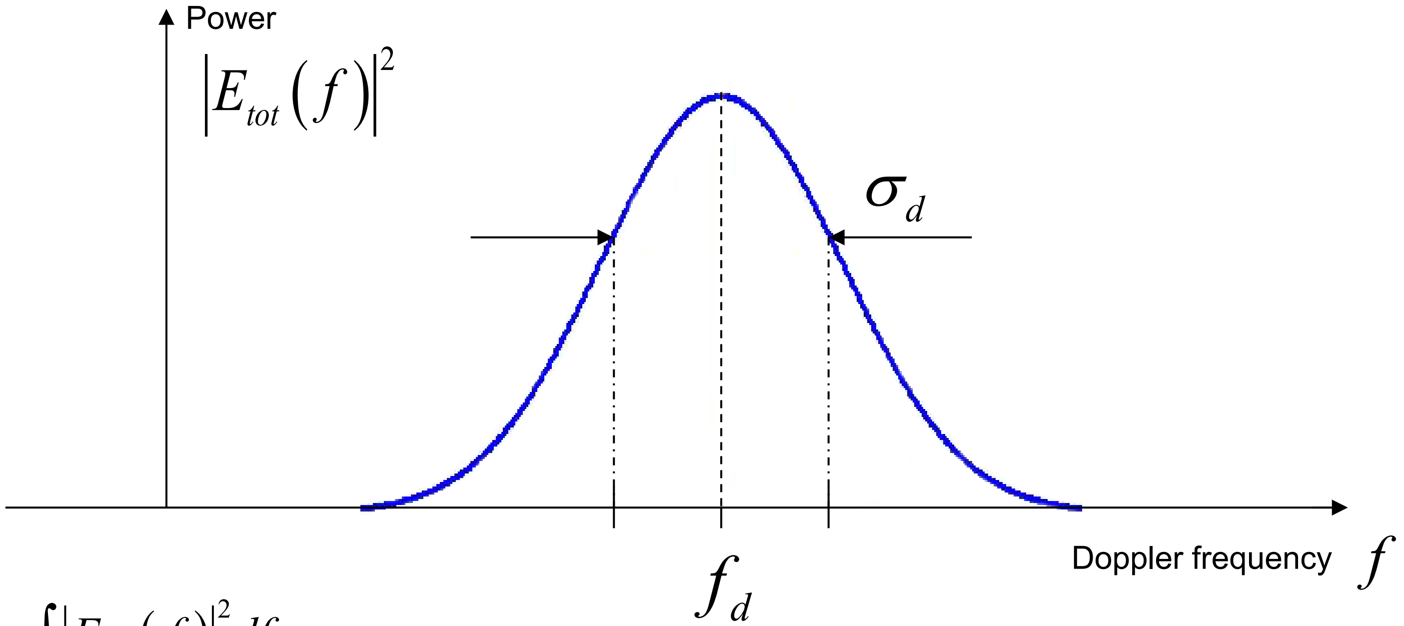
$$E_{tot}(f_m) = \frac{1}{T}\int_{t}^{t+T} \alpha_m(t)\, dt$$

Mean amplitude of all Scatterers with Doppler Frequency *Fm* and velocity *Vm*



Basis of fourier transform

We obtain a spectrum of all frequencies

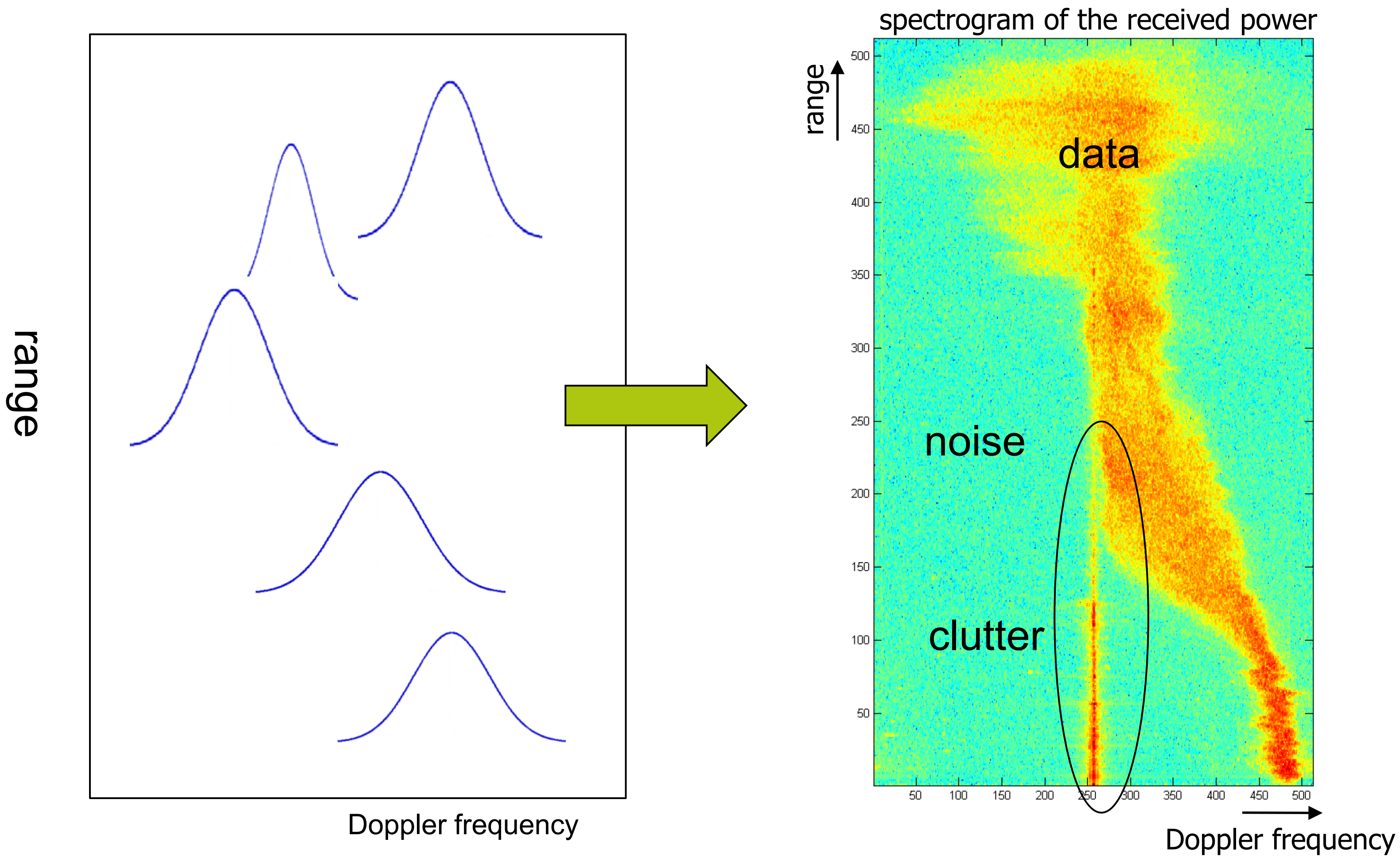


$$P_{tot} = \int |E_{tot}(f)|^2 df$$ ***Total power***

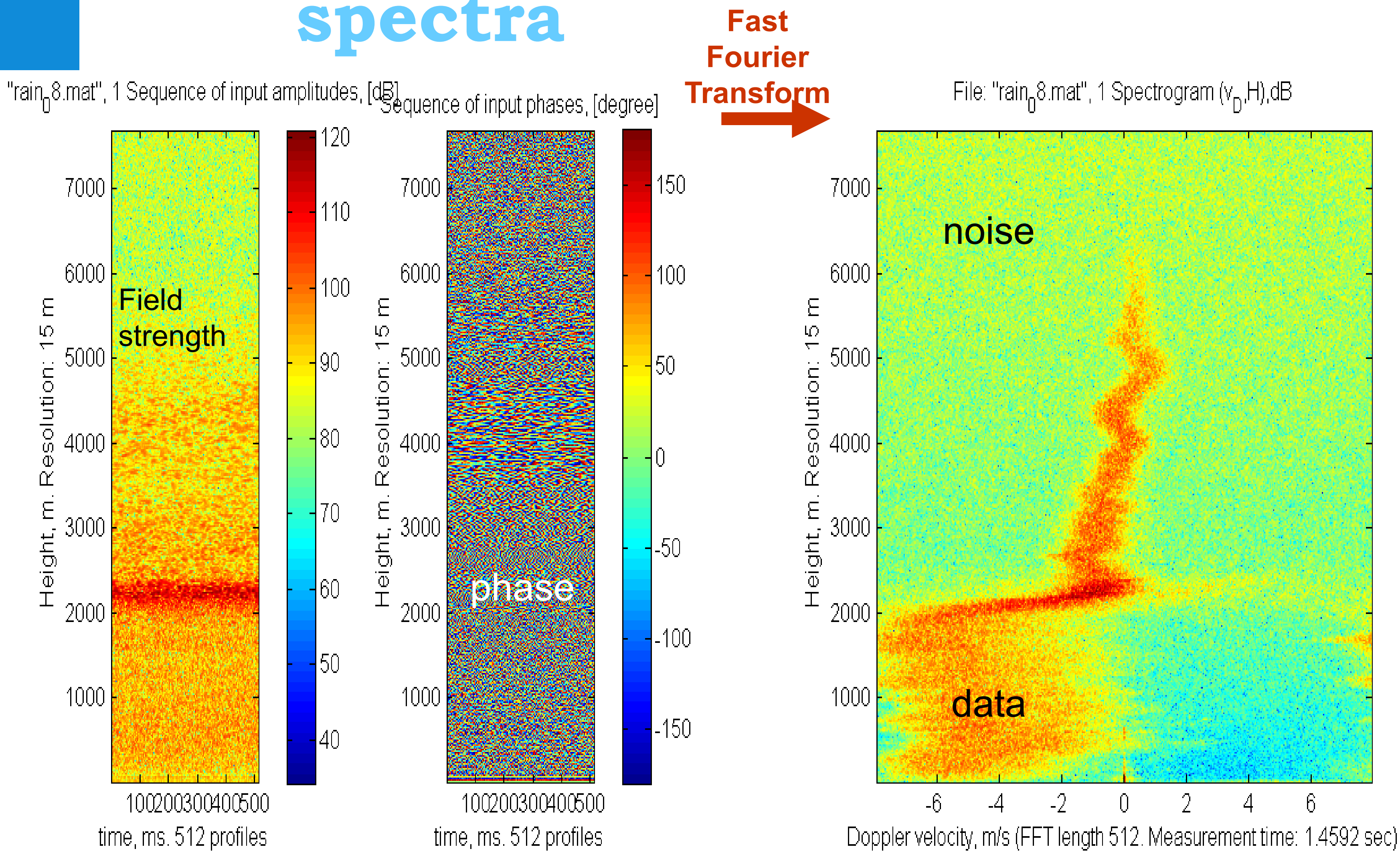
$$f_d = \frac{1}{P_{tot}} \int f |E_{tot}(f)|^2 df$$ ***Mean doppler frequency***

$$\sigma_d = \sqrt{\frac{1}{P_{tot}} \int (f - f_d)^2 |E_{tot}(f)|^2 df}$$ ***Doppler Width***

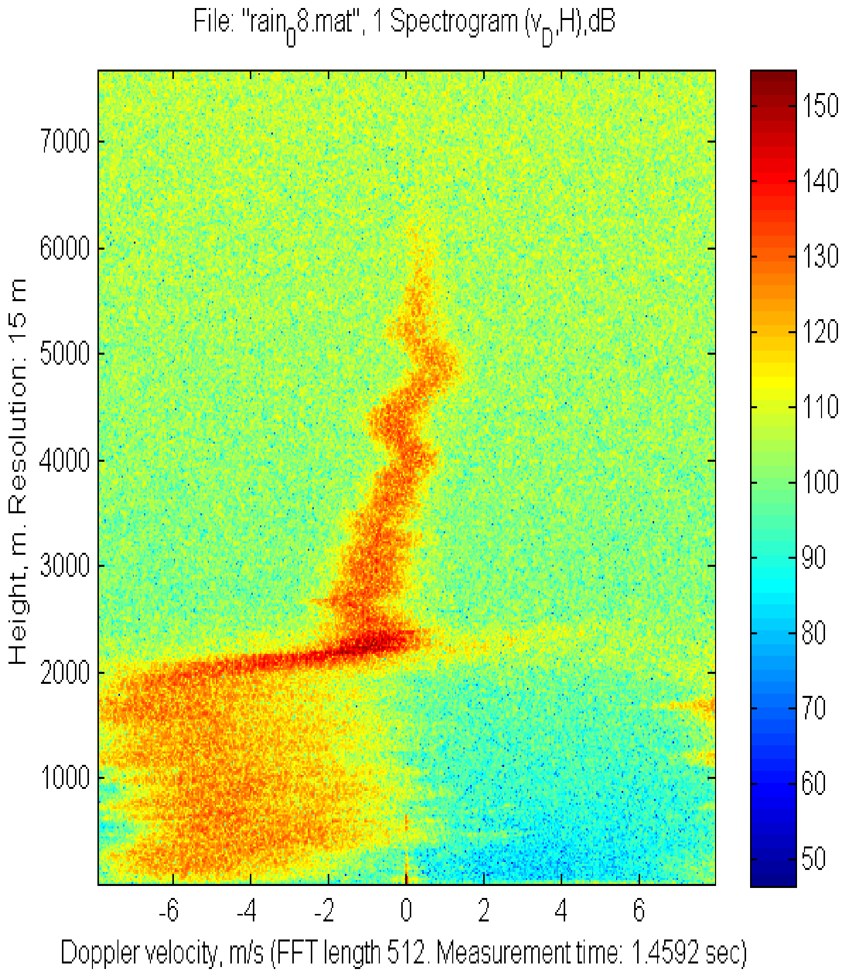
Repeat the procedure for all distances R



Raw radar signal and Doppler spectra



Spectrogram = Doppler spectra at every height



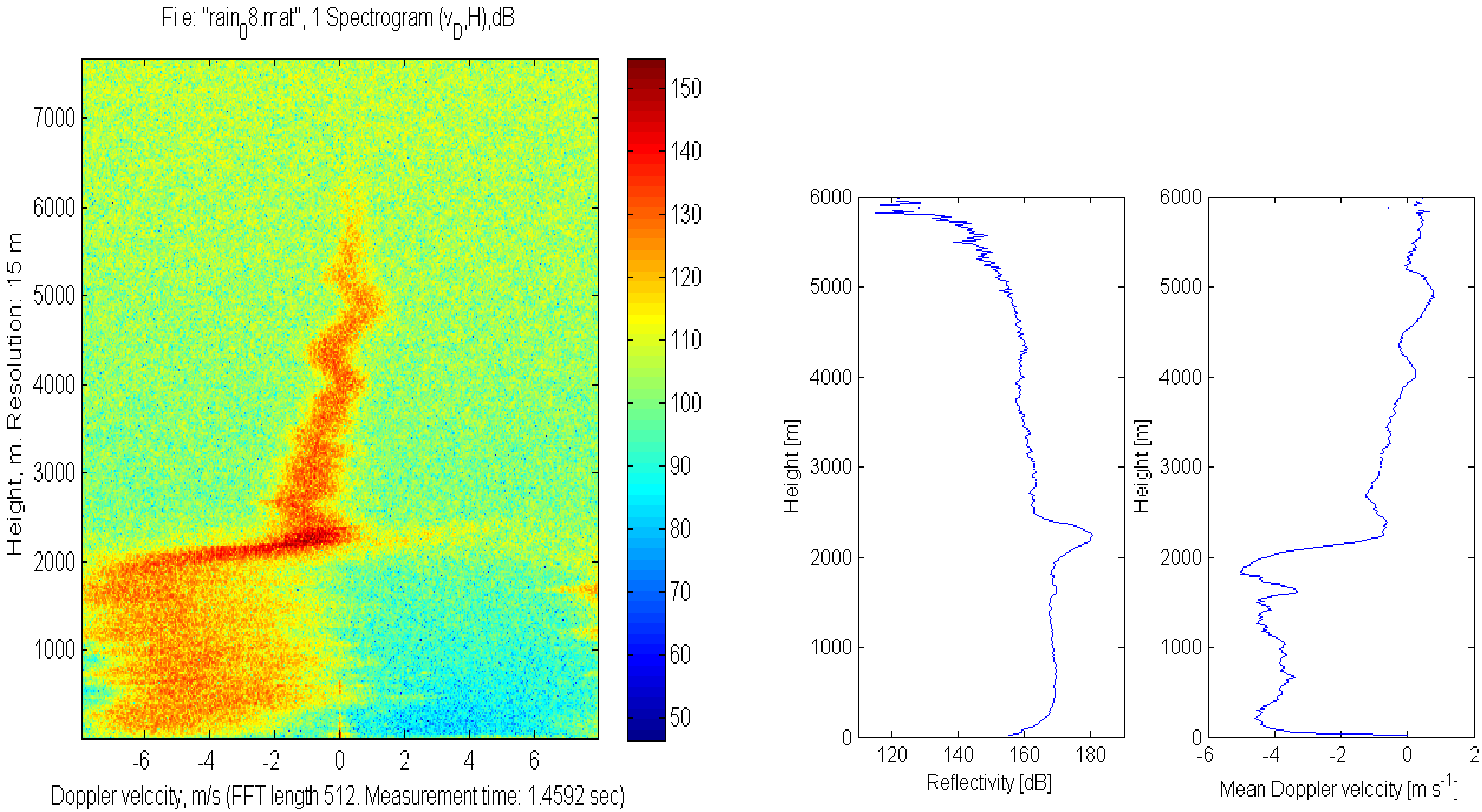
Further necessary steps

Remove the noise
Signal clipping at a certain threshold

Remove clutter
Filter around v=0 m/s

Calculate the total power, mean doppler Frequency, and the spectral width

From spectrogram to resulting profiles of power and mean



IDRA – TU Delft IRCTR Drizzle radar

CESAR – Cabauw Experimental Site for Atmospheric Research

IDRA is mounted on top of the 213 m high meteorological tower.

Specifications

- 9.475 GHz central frequency
- FMCW with sawtooth modulation
- transmitting alternately horizontal and vertical polarisation, receiving simultaneously the co- and the cross-polarised component
- 20 W transmission power
- 102.4 μs – 3276.8 μs sweep time
- 2.5 MHz – 50 MHz Tx bandwidth
- 60 m – 3 m range resolution
- 1.8° antenna half-power beamwidth

Reference

J. Figueras i Ventura: "Design of a High Resolution X-band Doppler Polarimetric Weather Radar", *PhD Thesis*, TU Delft, 2009. (online available at http://repository.tudelft.nl)

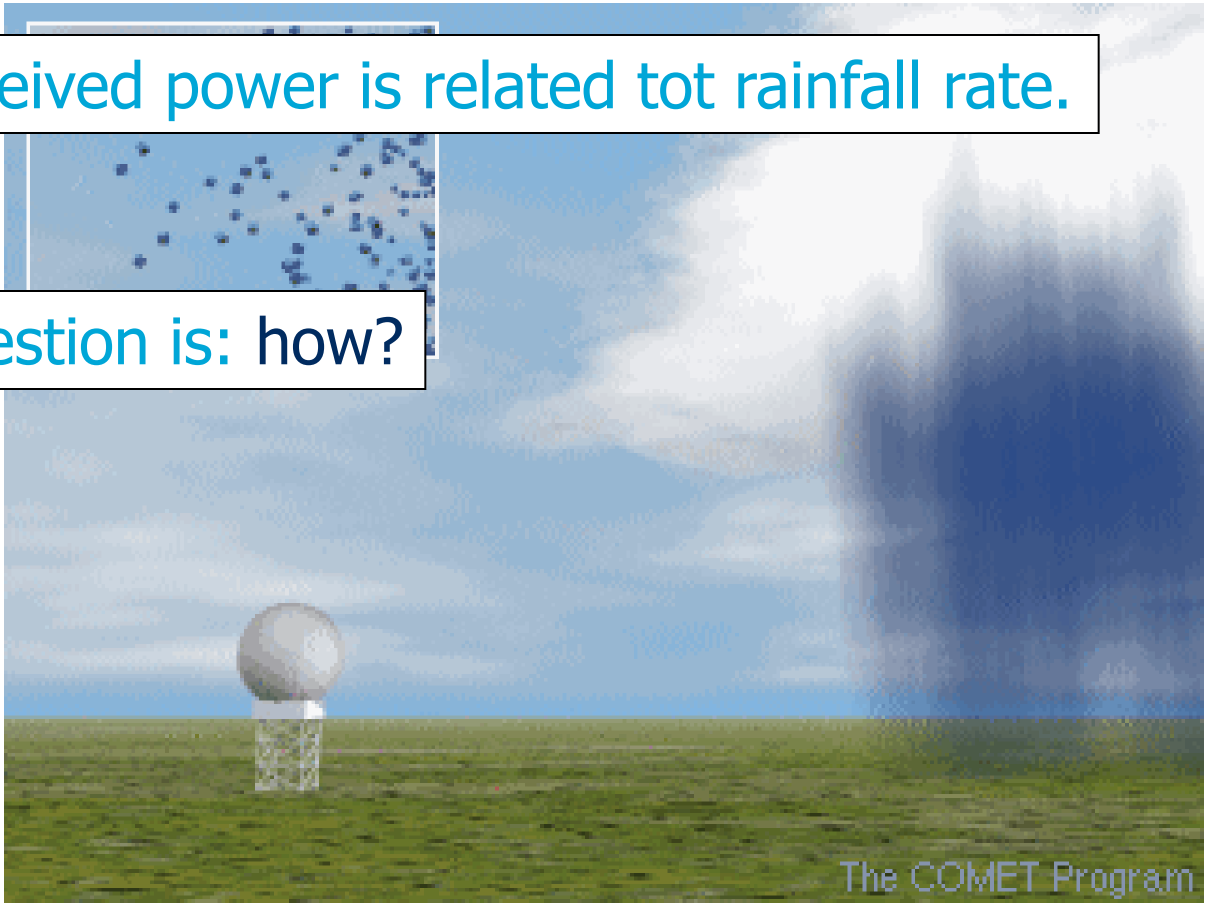
Near real-time display:
http://ftp.tudelft.nl/TUDelft/irctr-rse/idra

Processed and raw data available at:
http://data.3tu.nl/repository/collection:cabauw

Courtesy Otto

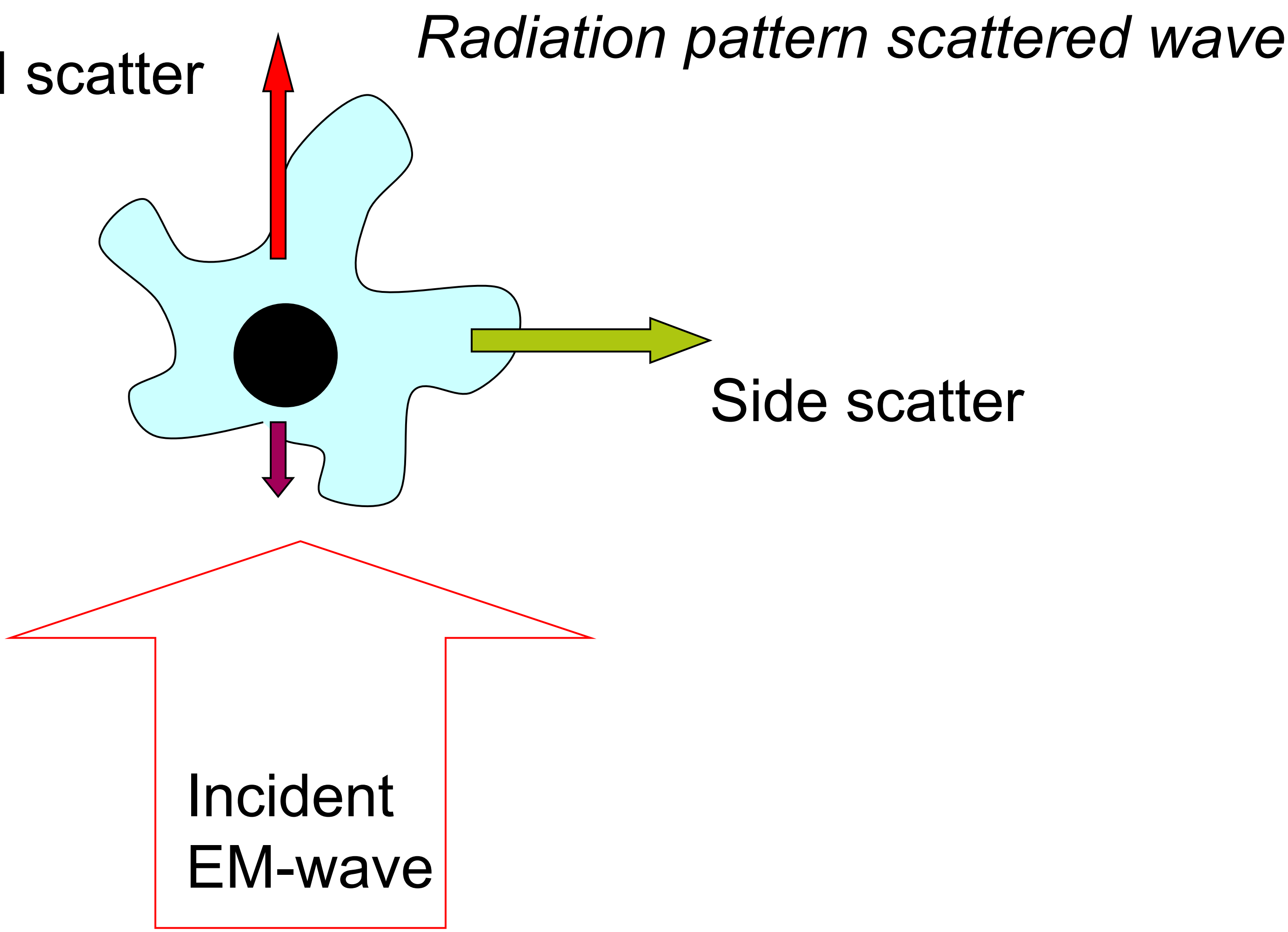
The received power is related tot rainfall rate.

The question is: how?

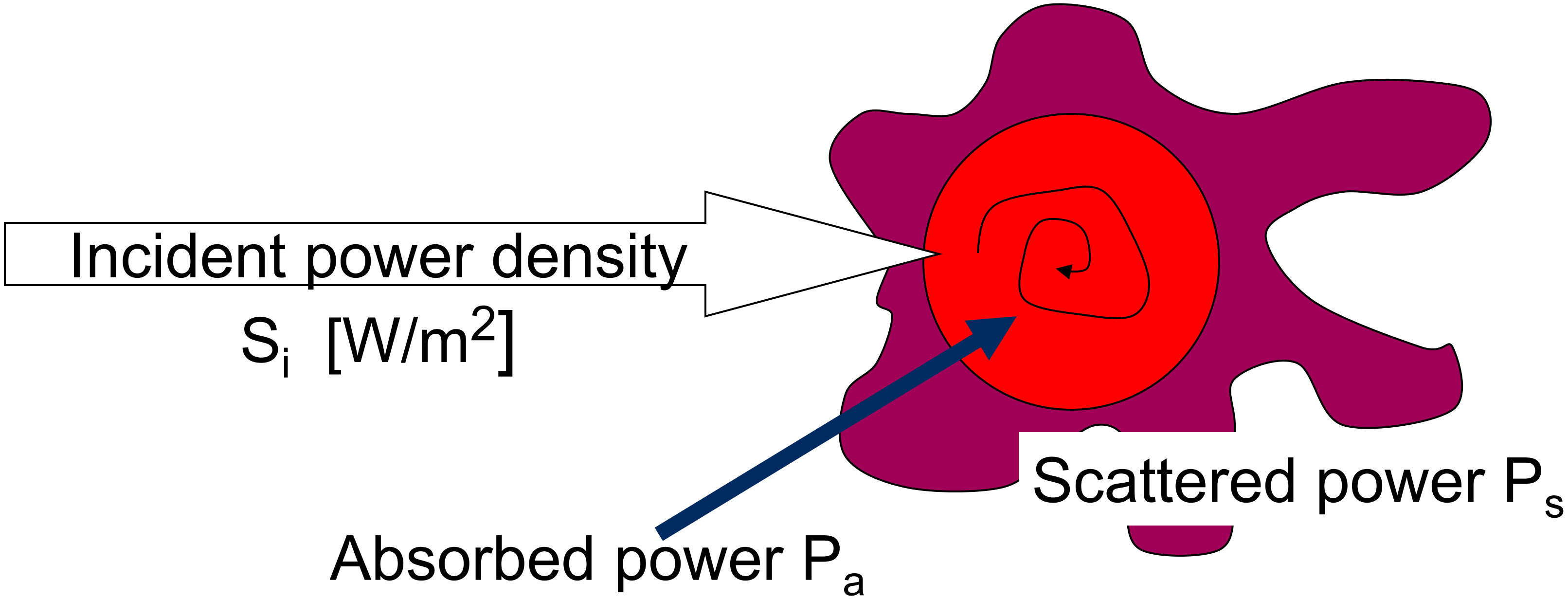


Source: www.everythingweather.com

First step: scattering by one particle



Definitions to describe scattering by one particle



Absorption cross-section $$\sigma_a = \frac{P_a}{S_i}$$

Scattering cross-section $$\sigma_s = \frac{P_s}{S_i}$$

Extinction cross-section $$\sigma_e = \sigma_a + \sigma_s$$

The definitions one step further:

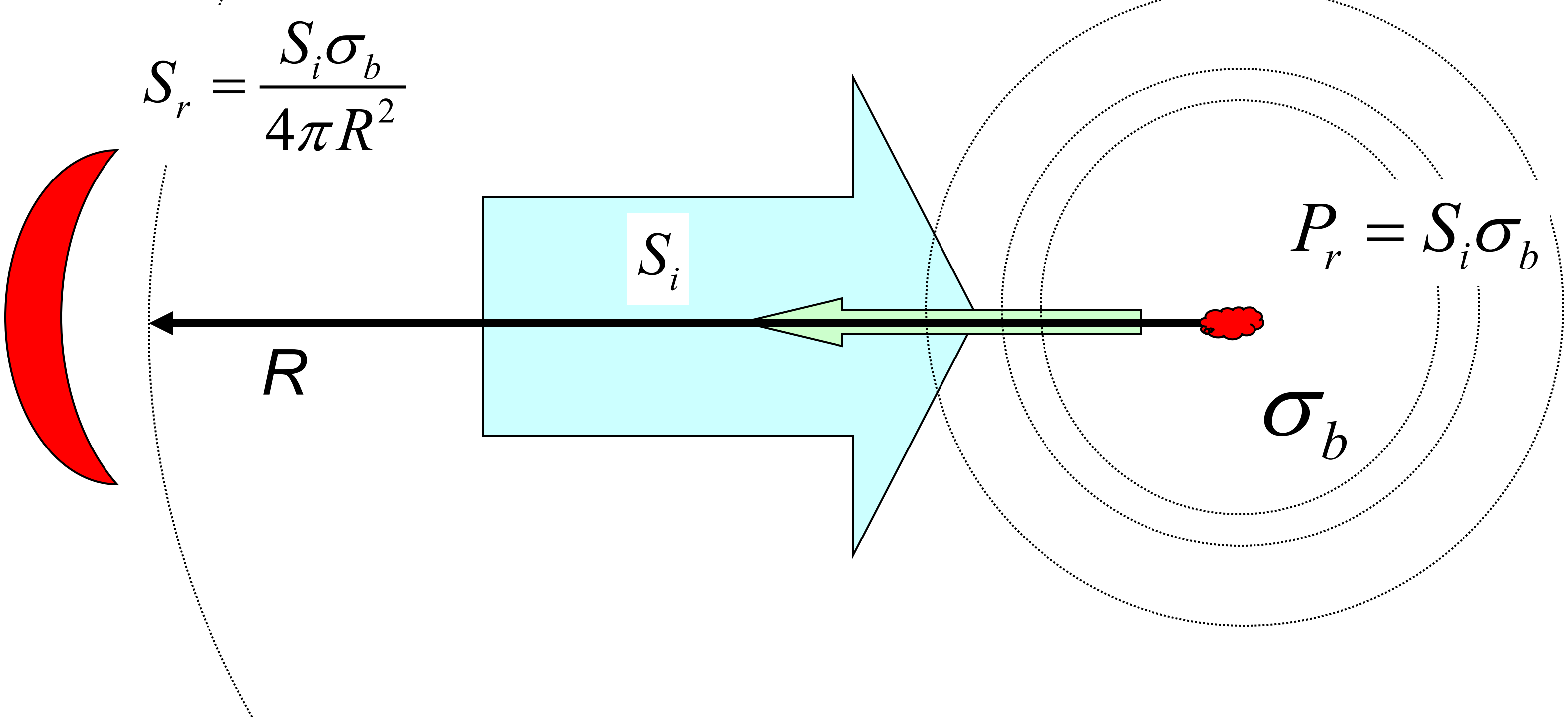
Efficiency factors

$$\zeta_s = \frac{\sigma_s}{A} \qquad \zeta_a = \frac{\sigma_a}{A} \qquad \zeta_e = \frac{\sigma_e}{A}$$

Geometric cross-section of particle

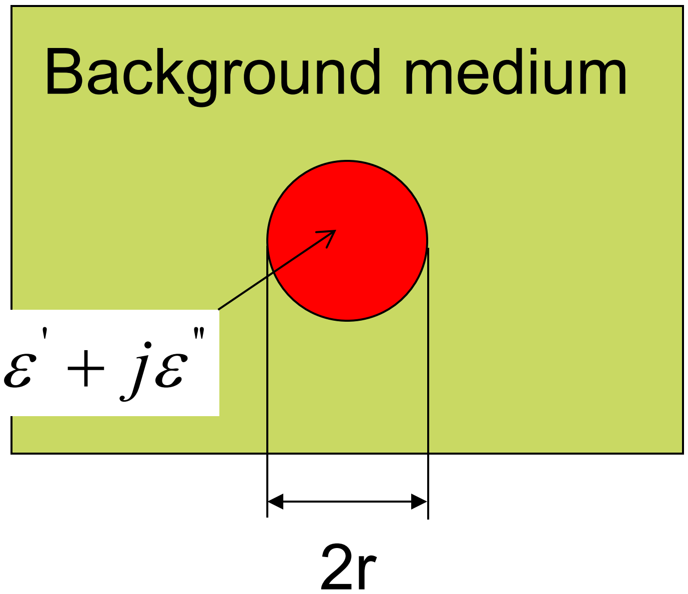
Scattering in all directions

Definition radar backscattering cross-section



Radar cross-section σ_b :
cross-section of equivalent isotropic radiator with power P_r

Scattering by a homogeneous dielectric sphere in a medium



$\varepsilon = \varepsilon' + j\varepsilon''$

permittivity ε

refractive index n

wavelength in sphere λ

Normalized radius

$$\chi = \frac{2\pi r}{\lambda} = \frac{2\pi r}{\lambda_o}\sqrt{\varepsilon'}$$

$$\varepsilon = n^2$$

wavelength in background λ_0

Scattering by a sphere is given by the Mie-formulas:

$$\zeta_s(n,\chi) = \frac{2}{\chi^2}\sum_l^{\infty}(2l+1)(|a_l|^2 + |b_l|^2)$$

$$\zeta_e(n,\chi) = \frac{2}{\chi^2}\sum_l^{\infty}(2l+1)(\mathrm{Re}(a_l + b_l))$$

$$\zeta_a(n,\chi) = \zeta_e(n,\chi) - \zeta_s(n,\chi)$$

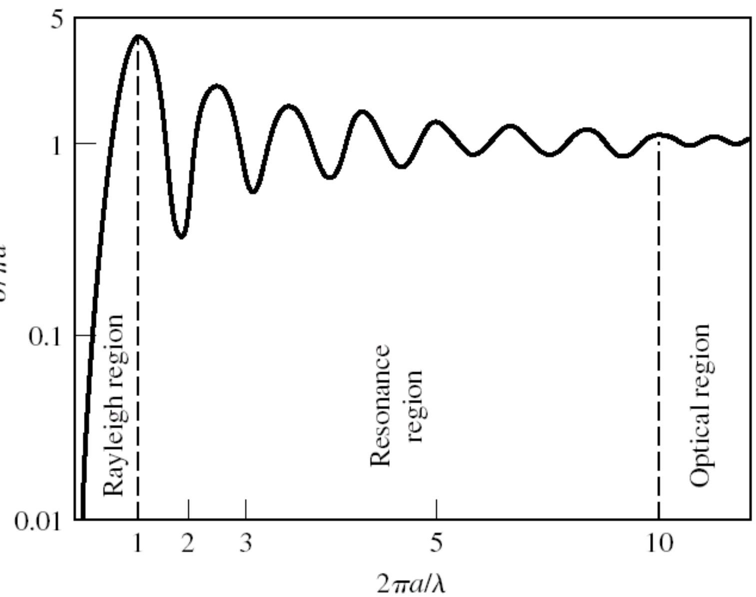
a_l, b_l :
Bessel, Hankel functions
depending on size and permittivity

$$\frac{\sigma_b}{\pi r^2} = \zeta_e(n,\chi) = \frac{1}{\chi^2}\left|\sum_l^{\infty}(-1)^l(2l+1)(a_l - b_l)\right|^2$$

Example of radar Cross Section σ

Monostatic radar cross section of a conducting sphere:

normalised radar cross section

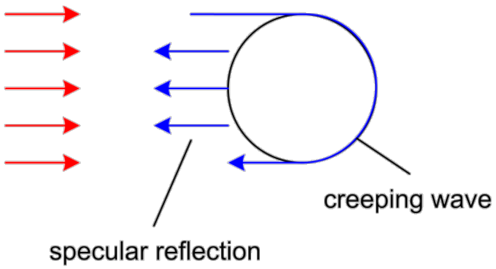


electrical size

a .. radius of the sphere
λ .. wavelength

Rayleigh region: a << λ

Resonance / Mie region:



Optical region: a >> λ

Figure: D. Pozar, “Microwave Engineering”, 2nd edition, Wiley.

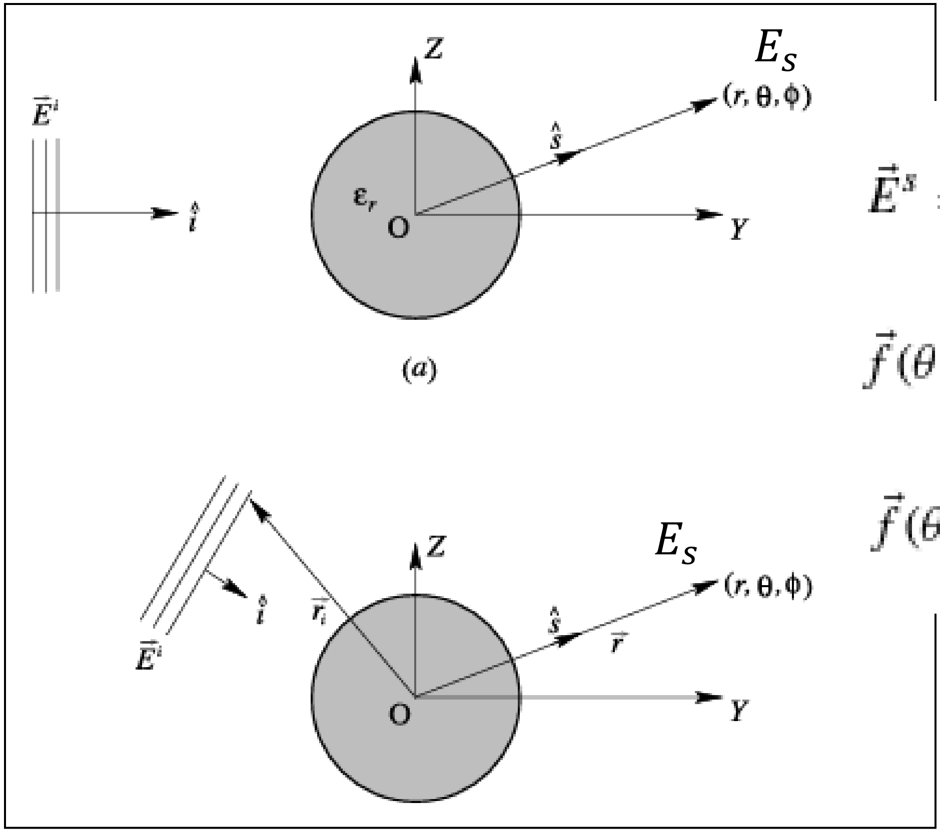
The Mie-formulation is exact for all particle sizes and wavelengths
but quite intractable,
therefore: approximations!

Most common: rayleigh approximation

Particle small compared to wavelength
Small phase shift of wave inside particle

$$\left| n\chi \right| \ll 1$$

Rayleigh scattering by a sphere



$$\vec{E}^s = \vec{f}(\theta,\phi)\frac{e^{-jk_0 r}}{r}$$

$$\vec{f}(\theta,\phi) = \frac{k_0^2}{4\pi}\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)}3V\left[\vec{E}^i - \hat{r}(\hat{r}\cdot\vec{E}^i)\right]$$

$$\vec{f}(\theta,\phi) = \frac{k_0^2}{4\pi\varepsilon_0}\left[\vec{p} - \hat{r}(\hat{r}\cdot\vec{p})\right]$$

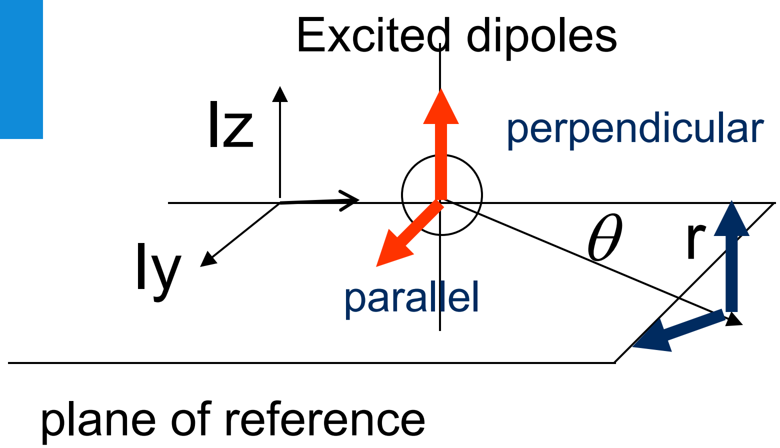
The scattered field depends on the looking direction and polarization of the incoming field

$\vec{p} = \alpha\vec{E}^i$ $\quad \alpha = 3\varepsilon_0 V(\varepsilon_r - 1)/(\varepsilon_r + 2),$

↓ polarizibility

Dipole moment: the sphere acts like a dipole

Physical interpretation Rayleigh approximation



$$I_{parallel} = \frac{I_y k^4 |\alpha|^2 \cos^2(\theta)}{r^2}$$

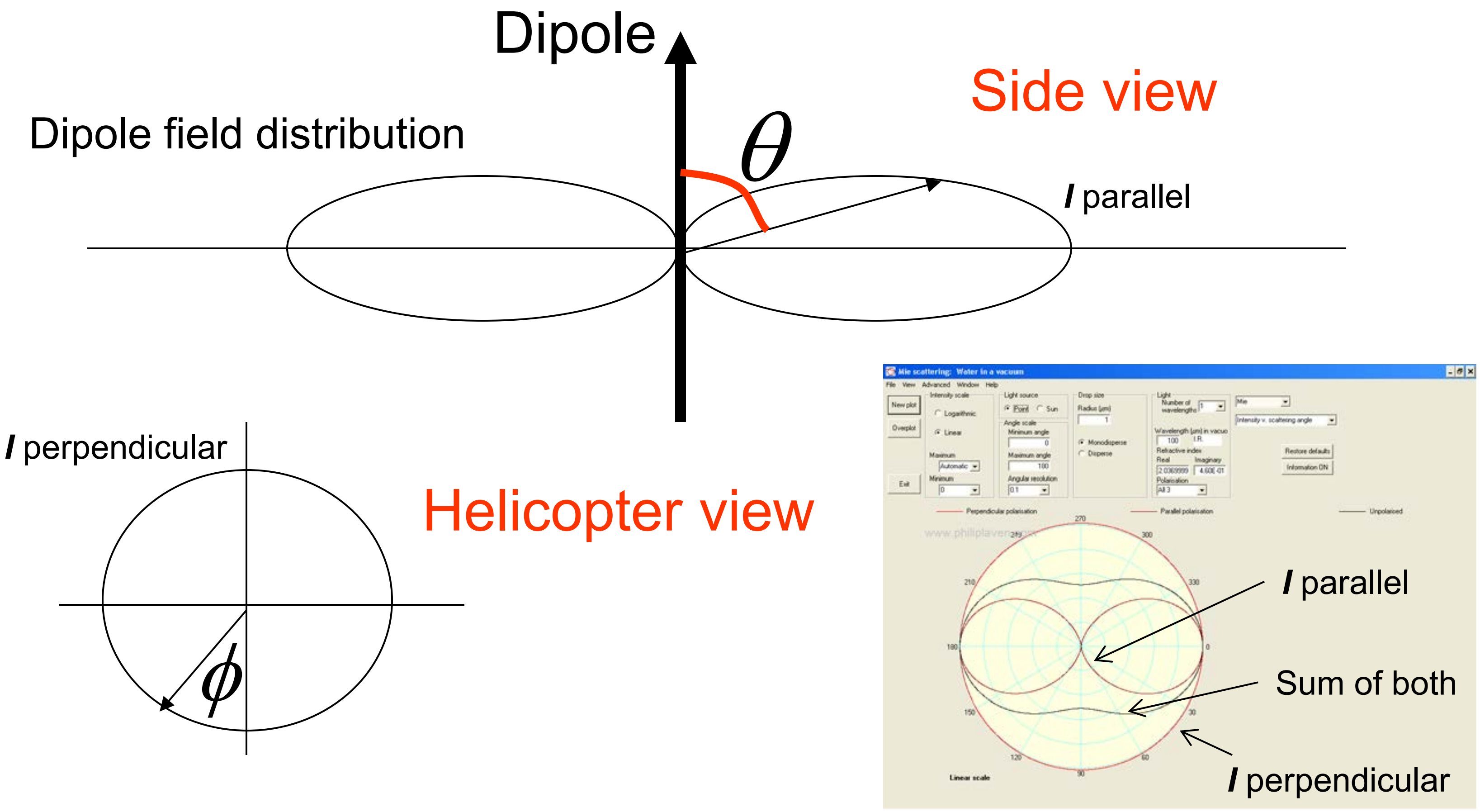
$$I_{perpendicular} = \frac{I_z k^4 |\alpha|^2}{r^2}$$

α Polarizibility of particle (~volume)
k Wave number

Unpolarized wave (*Iz=Iy=Io*):

$$I_{unpolarized} = \frac{I_{parallel} + I_{perpendicular}}{2} = \frac{I_o k^4 |\alpha|^2 \left(1+\cos^2(\theta)\right)}{r^2}$$

Physical interpretation Rayleigh approximation



The Rayleigh fields lead to the following cross-sections:

$$\sigma_s = \frac{2\lambda^2}{3\pi}\chi^6 |K|^2 = \frac{2}{3}\frac{\pi^5 |K|^2}{\lambda^4} D^6$$

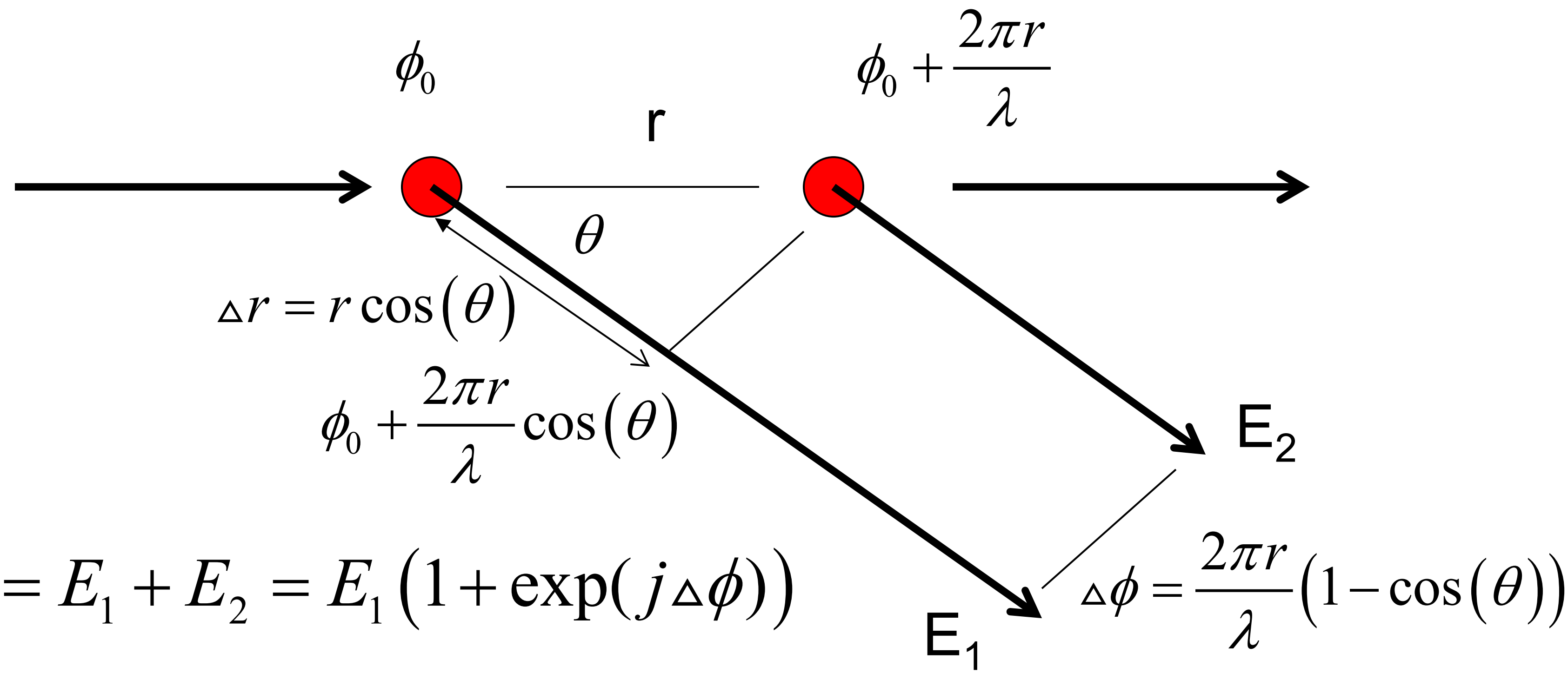
$$\sigma_a = \frac{\lambda^2}{\pi^3}\chi^3 \,\mathrm{Im}(-K) = \frac{\pi^2 D^3}{\lambda}\mathrm{Im}(-K)$$

$$\sigma_b = \lambda^2 \chi^6 |K|^2 = \frac{\pi^5 |K|^2}{\lambda^4} D^6$$

$$K = \frac{n^2 - 1}{n^2 + 2} = \frac{\varepsilon - 1}{\varepsilon + 2}$$

These cross-sections result from integration of the scattered-field intensities over space

The summation of fields from different scatterers



$$E_{tot} = E_1 + E_2 = E_1\left(1+\exp(j\triangle\phi)\right)$$

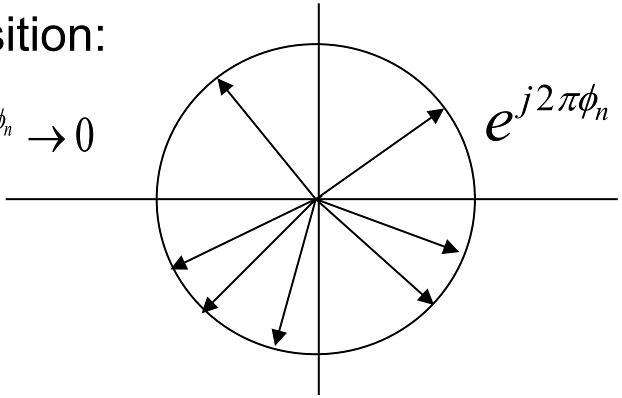
$$\theta = 0 \rightarrow E_{tot} = 2E_1$$

Independent of separation between particles

Forward scattering is always coherent: constructive interference
Scattering in other directions is always (partially) incoherent: (partially) destructive interference: less signal

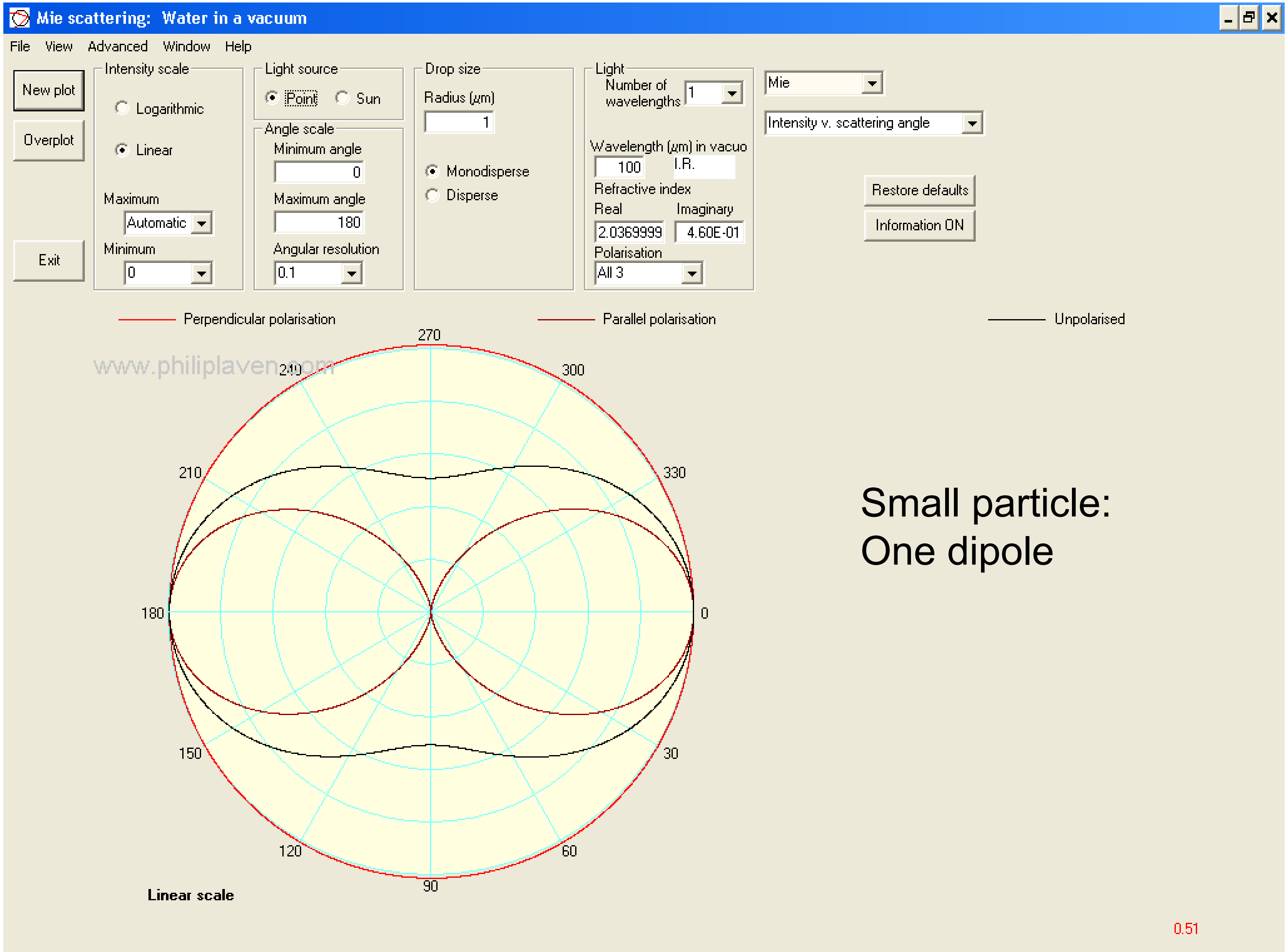
In case of many dipoles with random position:

$$\sum_{n}^{N} e^{j2\pi\phi_n} \to 0$$

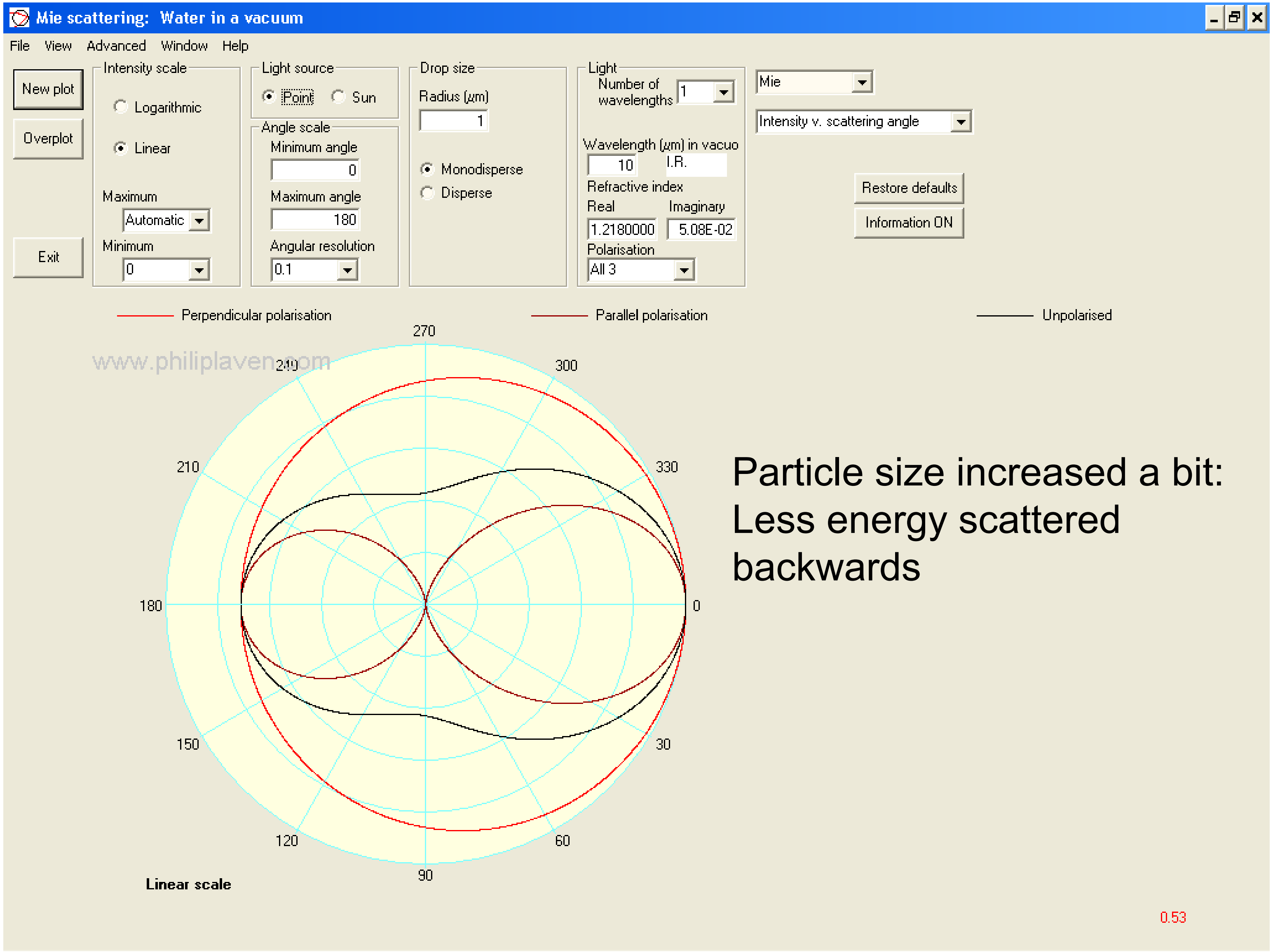


→ The scattering in the non-forward direction decreases, and becomes very small in the backward direction

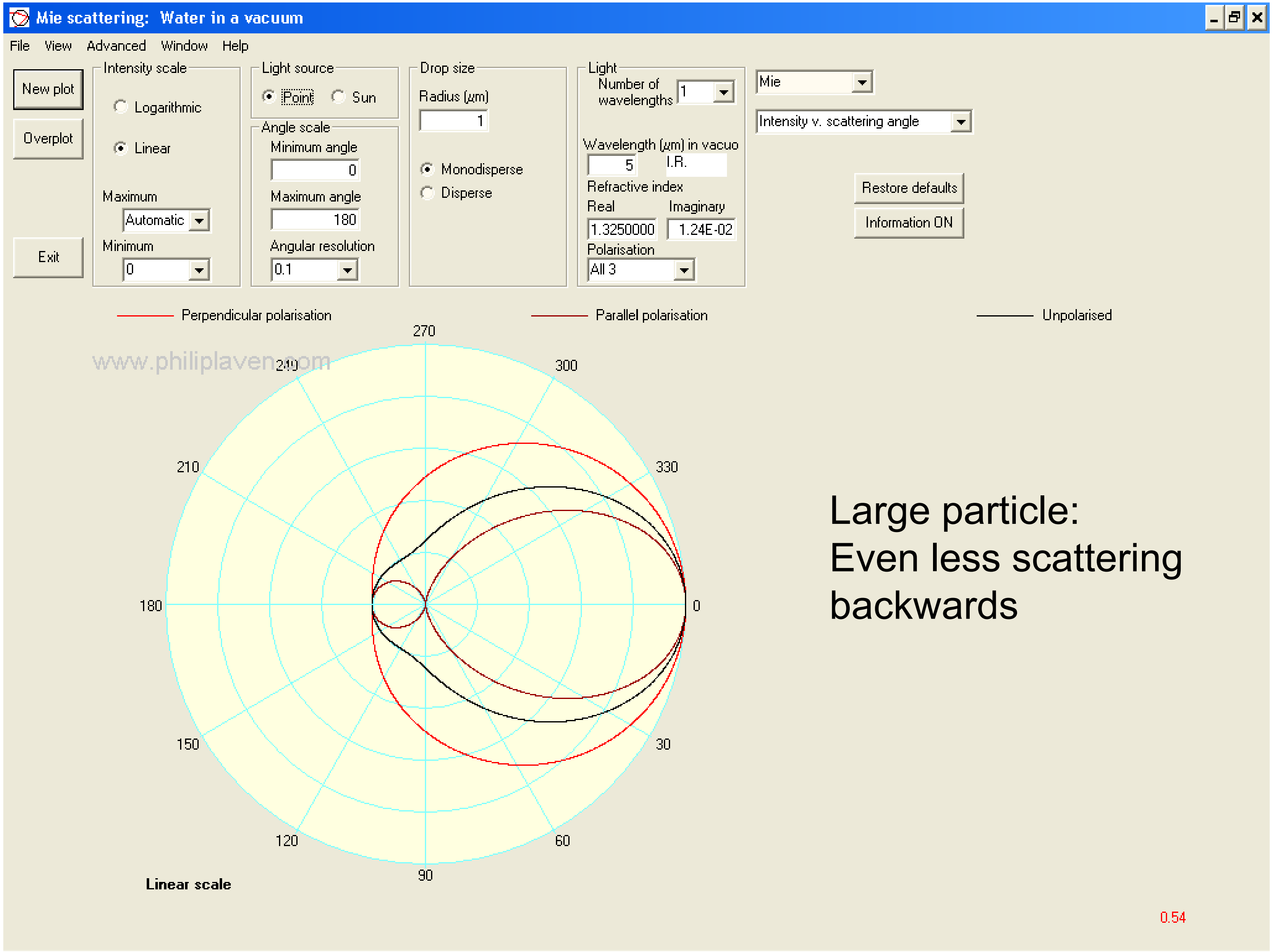
Now suppose we have a large sphere, compared to the wavelength: we can regard the sphere as a collection of dipoles, which means we get a radiation pattern. The consequences can be seen in the following Examples>



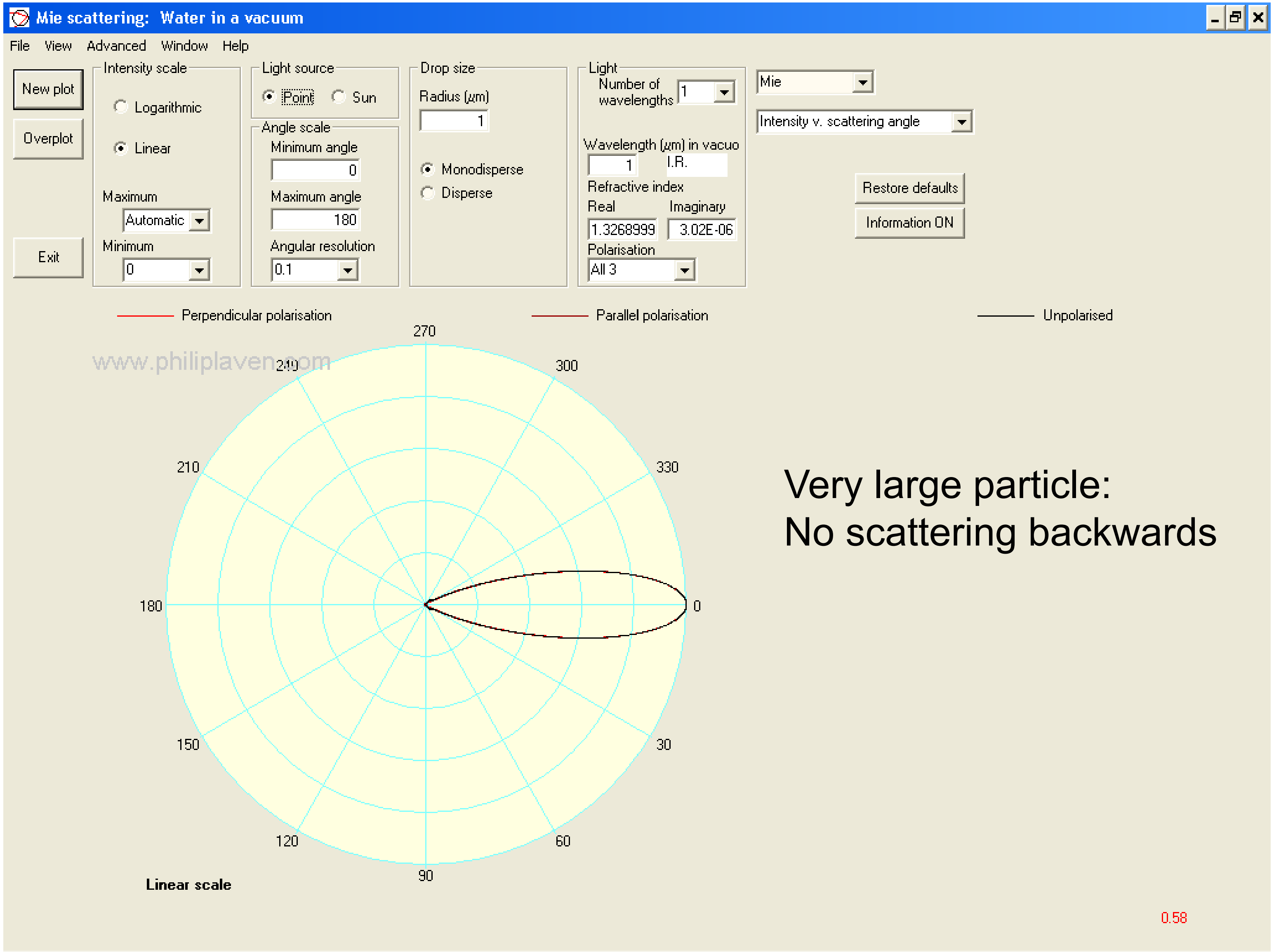
Small particle:
One dipole



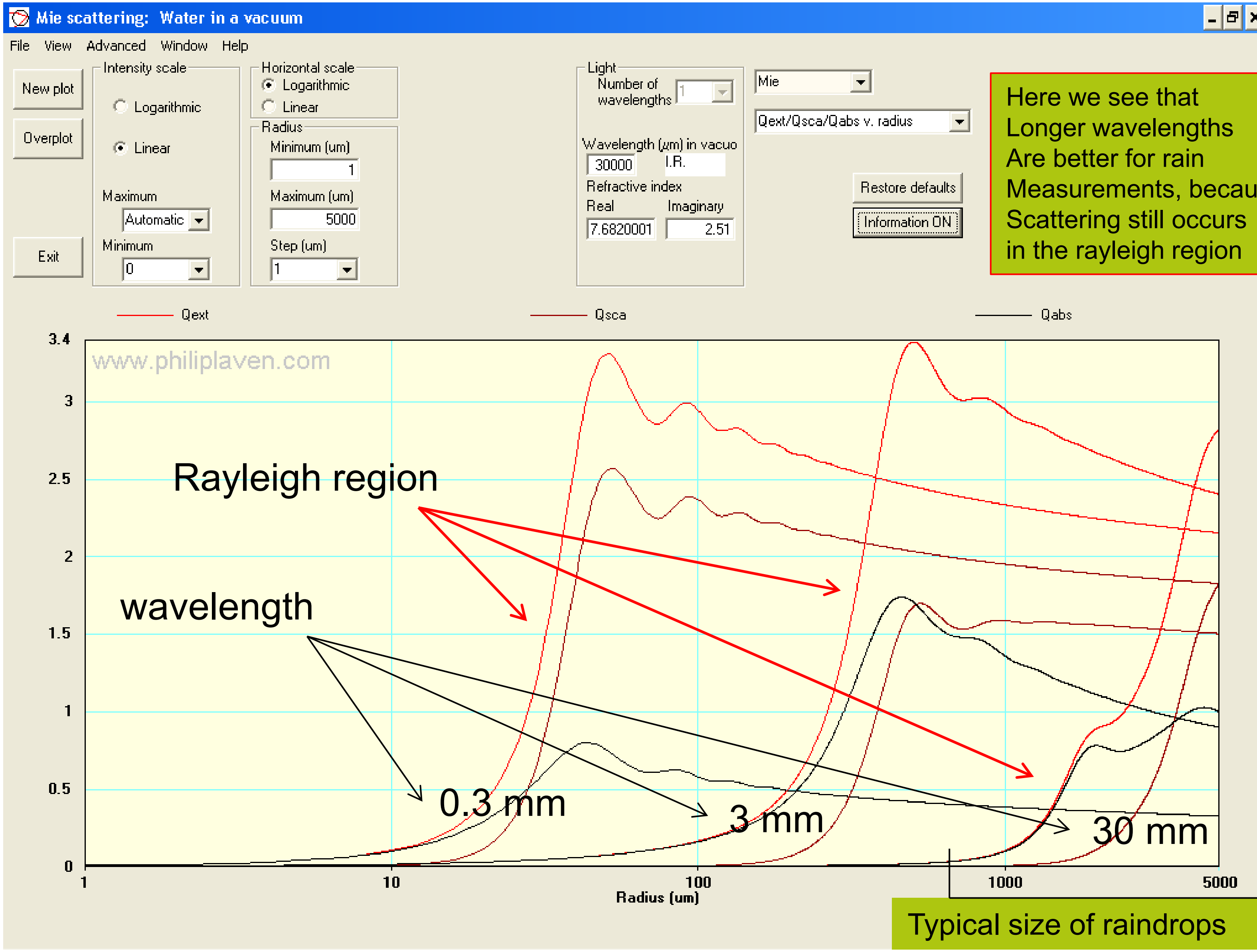
Particle size increased a bit: Less energy scattered backwards



Large particle:
Even less scattering backwards



Very large particle:
No scattering backwards



Here we see that
Longer wavelengths
Are better for rain
Measurements, because
Scattering still occurs
in the rayleigh region

Typical size of raindrops

Radar Equation for Weather Radar

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^3 R^4} \cdot \frac{\pi R^2 \theta^2 c}{16 \ln 2 B} \cdot \sum_{\text{unit volume}} \sigma_i$$

$$P_r = \frac{\pi^3 P_t G_t G_r \theta^2 c}{1024 \ln 2 B \lambda^2} \cdot |K|^2 \sum_{\text{unit volume}} D_i^6 \cdot \frac{1}{R^2}$$

radar constant

radar reflectivity factor *z*, purely a property of the observed precipitation

Material property

Radar Reflectivity Factor z

$$z = \sum_{\text{unit volume}} D_i^6 \left(\frac{\text{mm}^6}{\text{m}^3}\right)$$

→ spans over a large range; to compress it into a smaller range of numbers, a logarithmic scale is preferred

$$Z = 10\log_{10}\left(\frac{z}{1\text{mm}^6/\text{m}^3}\right)(\text{dBZ})$$

To measure the reflectivity by the weather radar, we need to:

- know the radar constant *C*,
- measure the mean received power P_r,
- measure the range *R*,
- and apply the radar equation for weather radars:

$$z = P_r C R^2$$

Raindrop-Size Distribution *N(D)*

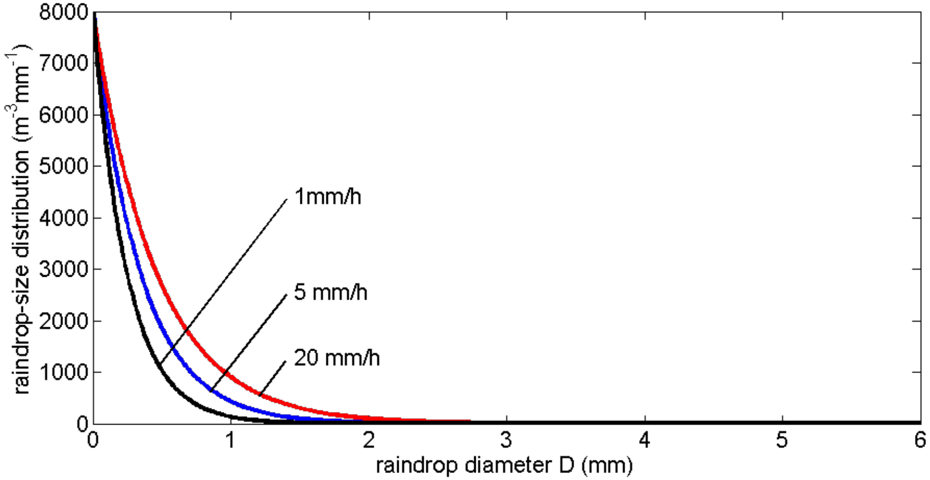
$$z = \sum_{\text{unit volume}} D_i^6 = \int_0^\infty D^6 N(D)\,dD = \frac{N_0}{\Lambda^7}\cdot 6!$$

where *N(D)* is the raindrop-size distribution that tells us how many drops of each diameter *D* are contained in a unit volume, i.e. 1m^3.

Often, the raindrop-size distribution is assumed to be exponential:

$$N(D) = N_0 \exp(-\Lambda D)$$

Intercept parameter (m^{-3}mm^{-1}) slope parameter (mm^{-1})



Marshall and Palmer (1948):

N_0 = 8000 m^{-3}mm^{-1}
Λ = $4.1\cdot R^{-0.21}$

with the rainfall rate *R* (mm/h)

The raindrop size distribution is a model we need to interpret the radar received power in terms of rainfall rate.

$$N(D) = N_0 \exp(-\Lambda D)$$

Intercept parameter ($m^{-3}mm^{-1}$)   slope parameter (mm^{-1})

In case of the Marshall-Palmer distribution we fix No, and let the slope parameter vary. We therefore need one radar observable to estimate the slope parameter.

Reflectivity – Rainfall Rate Relations

reflectivity (mm^6m^{-3}) $$z = \int_D D^6 N(D)dD$$

liquid water content (mm^3m^{-3}) $$\mathrm{LWC} = \frac{\pi}{6}\int_D D^3 N(D)dD$$

raindrop volume

rainfall rate ($mm\ h^{-1}$) $$R = \frac{\pi}{6}\int_D D^3 v(D) N(D)dD$$

terminal fall velocity

$$\boldsymbol{v(D) = 9.65 - 10.3e^{-0.6D}}$$

→ the reflectivity measured by weather radars can be related to the liquid water content as well as to the rainfall rate:

power-law relationship $$z = aR^b$$

the coefficients *a* and *b* vary due to changes in the raindrop-size distribution or in the terminal fall velocity.

Often used as a first approximation is *a* = 200 and *b* = 1.6

Importance of knowing dropsizes

Drop Size	#/m^3	Z	Water Volume per cubic meter
1 mm	4096	36 dBZ	2144.6 mm^3
4 mm	1	36 dBZ	33.5 mm^3

Raincell: cylinder 10km diameter, 2 km height: 157079632679 m^3
Difference: 314159265 liter or the average annual water consumption of 3315 ‘standard’ households in The Netherlands

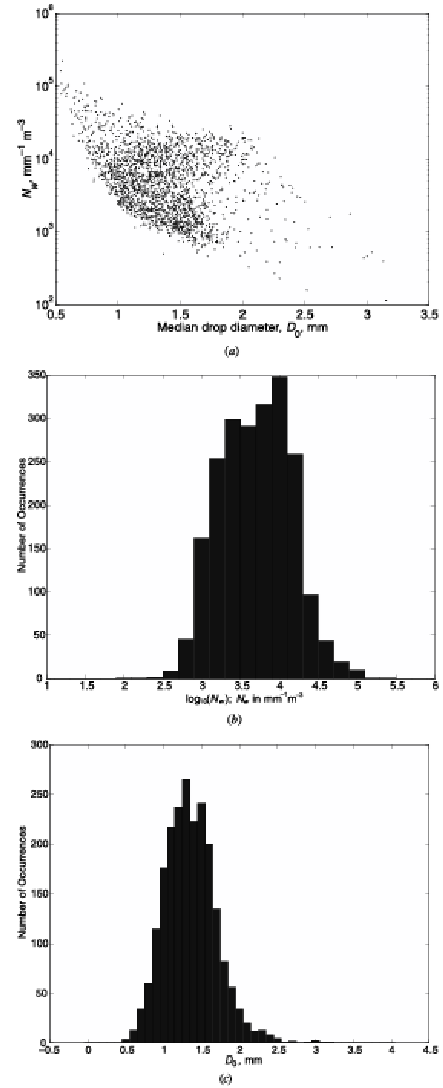
How constant is N(D)?

In our model we assumed a fixed No.
How correct is that?

Here we see a histogram of independent observations of No

Apparantly our model is not that accurate: No is not constant!

Histograms of dropsize measurements



So the question comes:
How can we measure No with the radar?

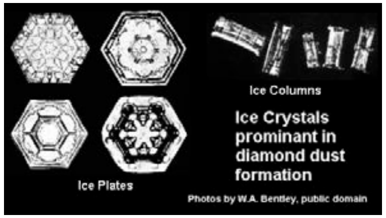
We can do this with polarization.

70

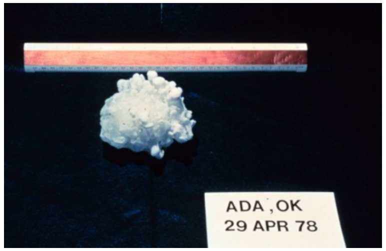
Can Polarimetry add Information?

➔ yes, **because hydrometeors are not spheres**

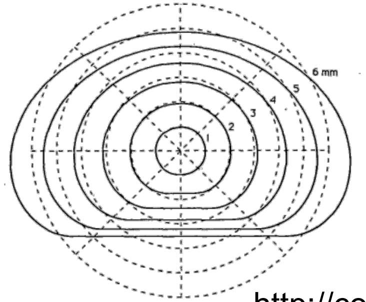
- ice particles



- hail



- raindrops

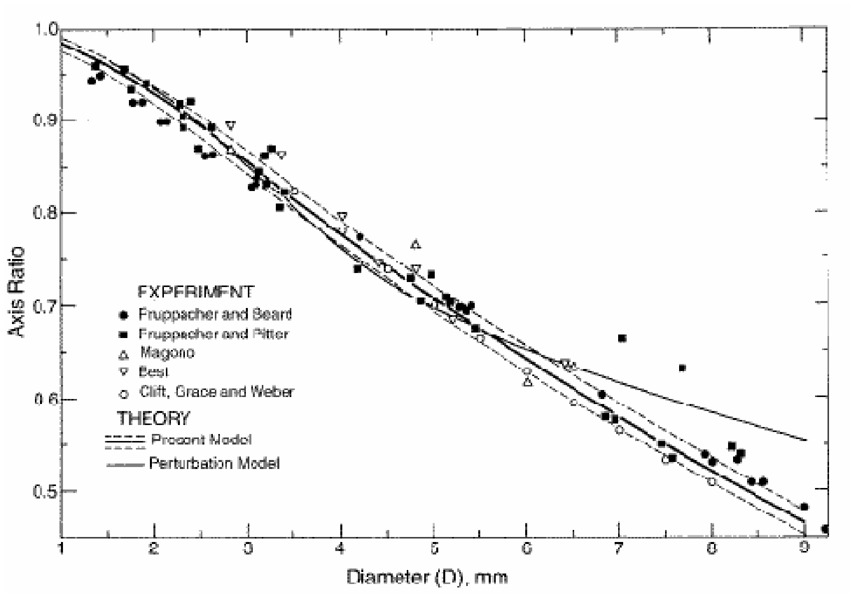


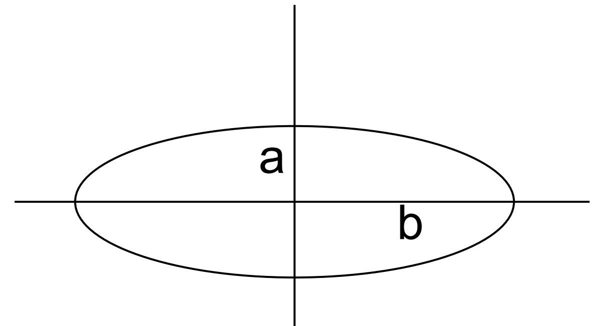
http://commons.wikimedia.org/wiki/Category:Hail

Observed shapes of raindrops

8.00 mm 7.35 5.8 5.30 3.45 2.70

Axial ratio of raindrops versus size



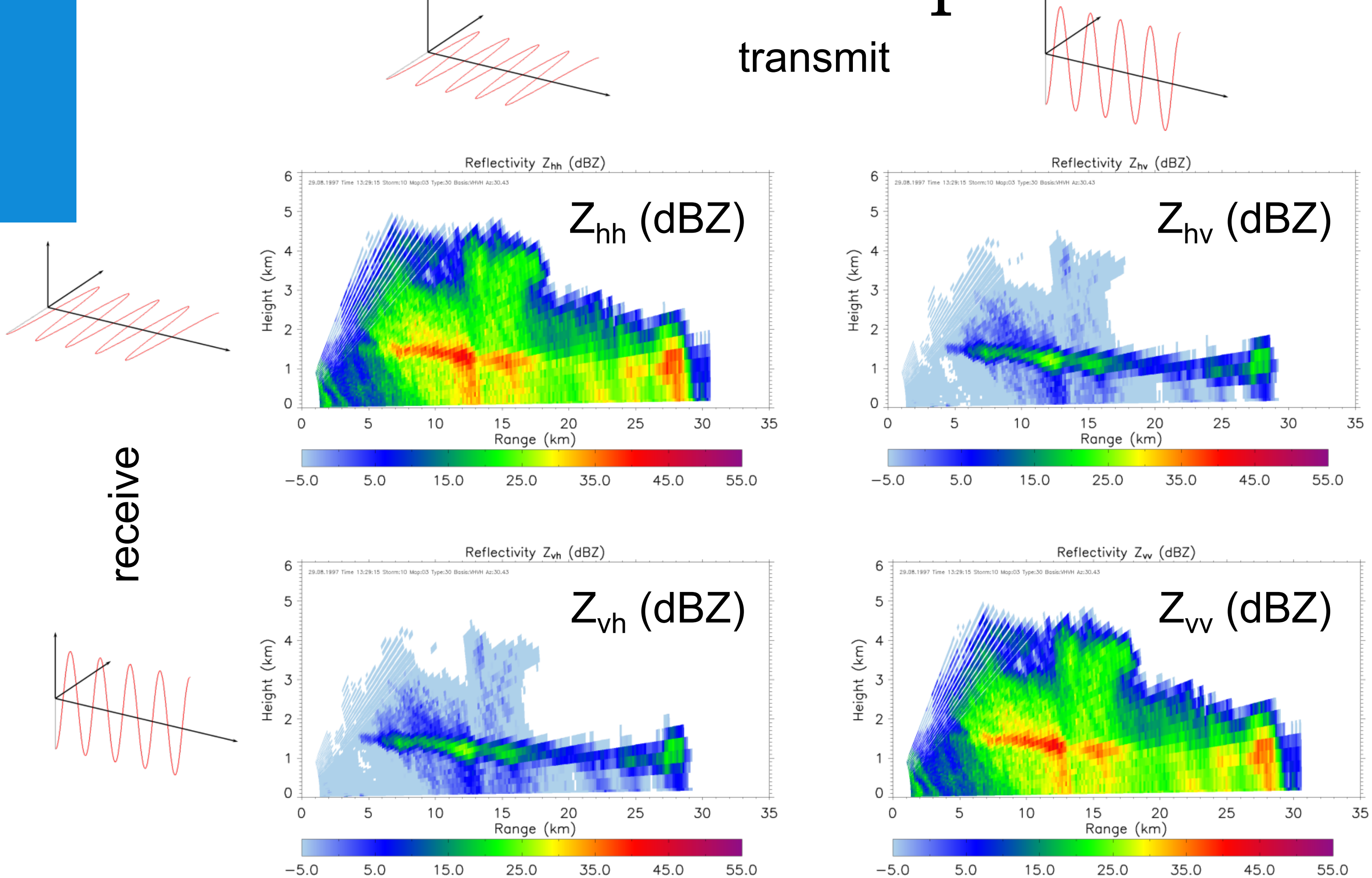


Axial ratio = a/b

When the particle becomes oblate or prolate,
the backscattering becomes
polarization dependent:

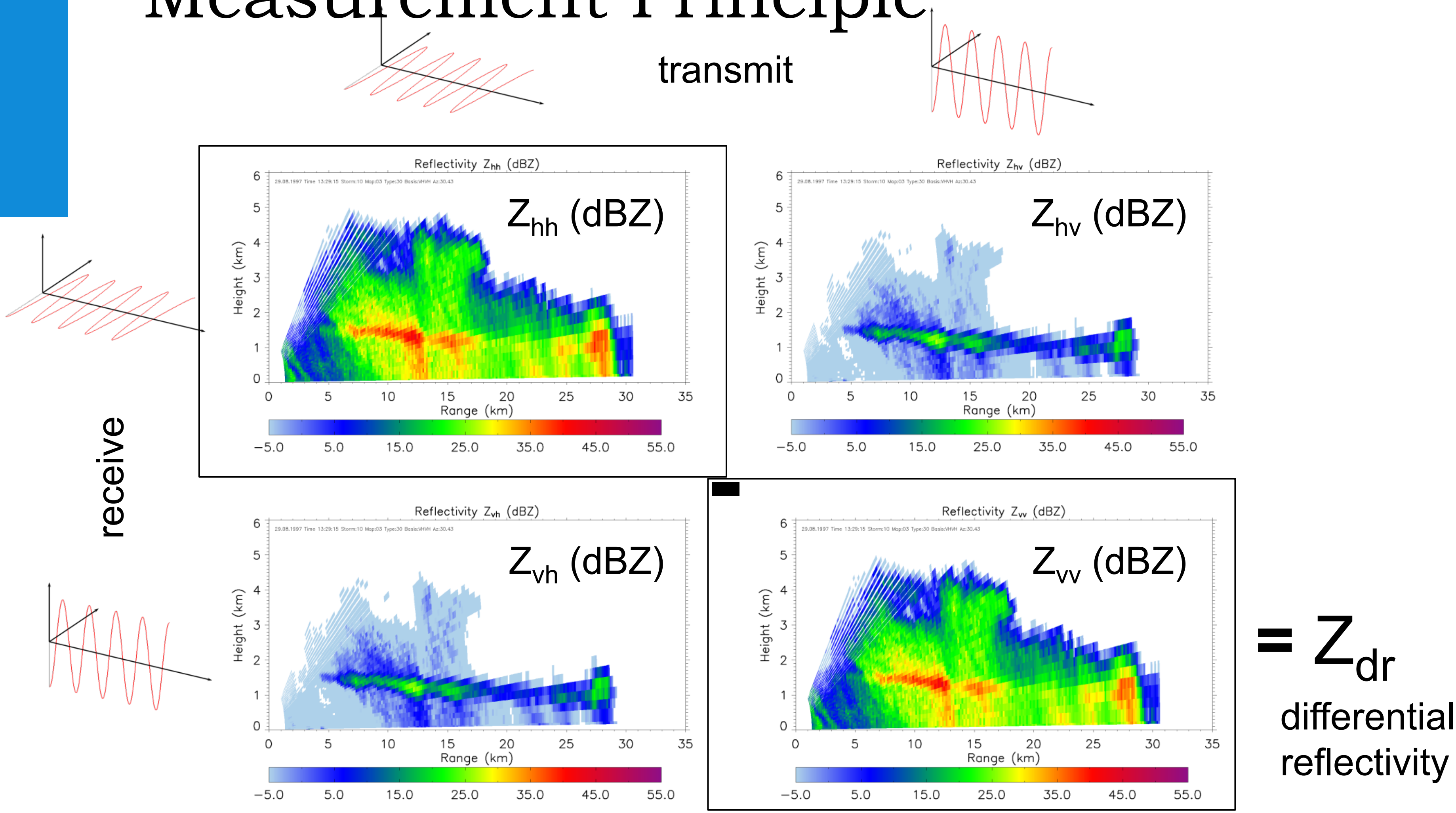
axial ratio < 1: HH > VV; axial ratio > 1: HH < VV; axial ratio = 1: HH = VV

Measurement Principle



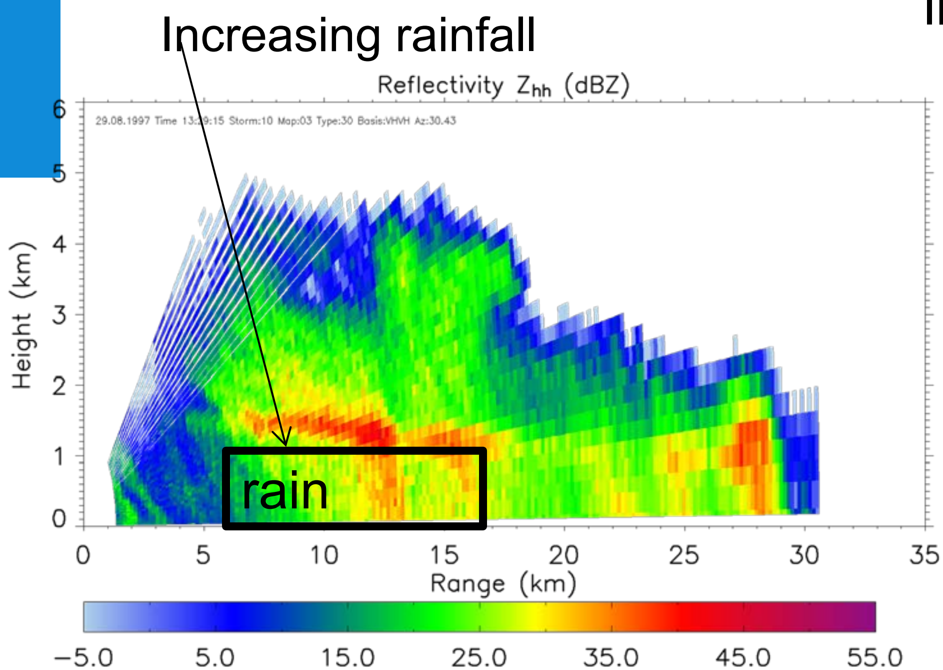
Data: POLDIRAD (DLR, Oberpfaffenhofen, Germany), Prof. Madhu Chandra

Measurement Principle

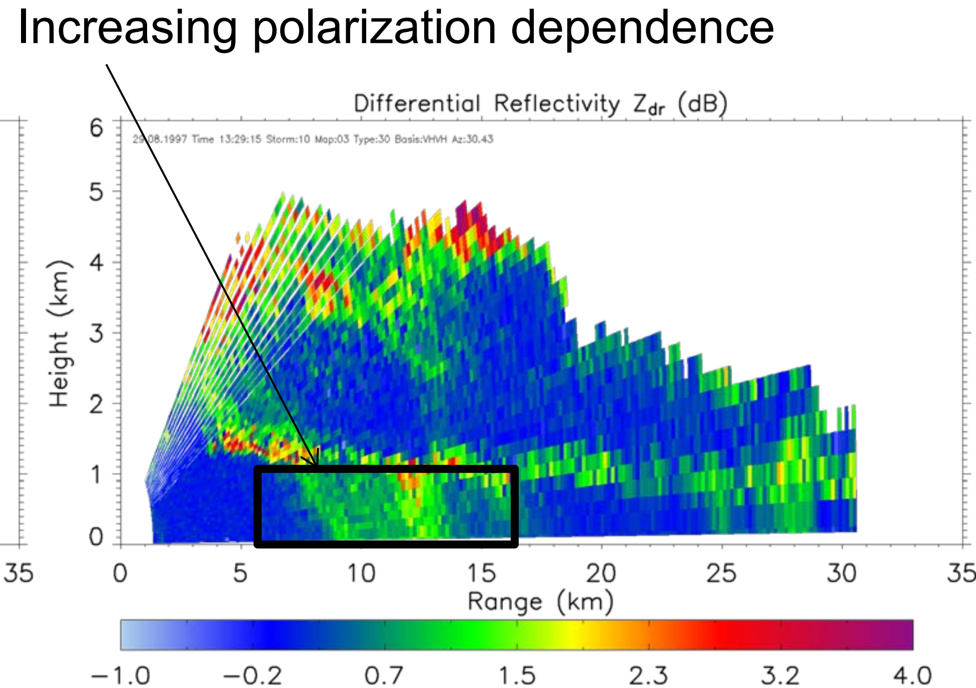


Data: POLDIRAD (DLR, Oberpfaffenhofen, Germany), Prof. Madhu Chandra

Changing raindrop shape



Reflectivity



Differential Reflectivity

$$Z_{hh} = 10 \log C R^2 \overline{P}_{hh} \left(\mathrm{dBZ}\right)$$

$$Z_{dr} = 10 \log \frac{\overline{P}_{hh}}{\overline{P}_{vv}} \left(\mathrm{dB}\right)$$

Data: POLDIRAD (DLR, Oberpfaffenhofen, Germany), Prof. Madhu Chandra

$$Z_{hh} = \int N(D)\sigma_{hh}(D)dD$$

$$Z_{dr} = \frac{Z_{hh}}{Z_{vv}} = \frac{\int N(D)\sigma_{hh}(D)dD}{\int N(D)\sigma_{vv}(D)dD}$$

$N(D)dD:$ Number of drops with diameter between D and D+dD

$\sigma_{hh,vv}:$ Radar cross-section for hh or vv polarization

$$N(D)dD = N_o e^{-3.67\frac{D}{D_o}} dD$$

$$Z_{hh} = \int N(D)\sigma_{hh}(D)dD$$

$$Z_{dr} = \frac{\int N(D)\sigma_{hh}(D)dD}{\int N(D)\sigma_{vv}(D)dD}$$

$$Z_{hh} = N_o \int \exp\left(-3.67\frac{D}{D_o}\right)\sigma_{hh}(D)\,dD$$

→ Do and Zhh gives No

$$Z_{dr} = \frac{\int \exp\left(-3.67\frac{D}{D_o}\right)\sigma_{hh}(D)\,dD}{\int \exp\left(-3.67\frac{D}{D_o}\right)\sigma_{vv}(D)\,dD}$$

→ Zdr gives Do

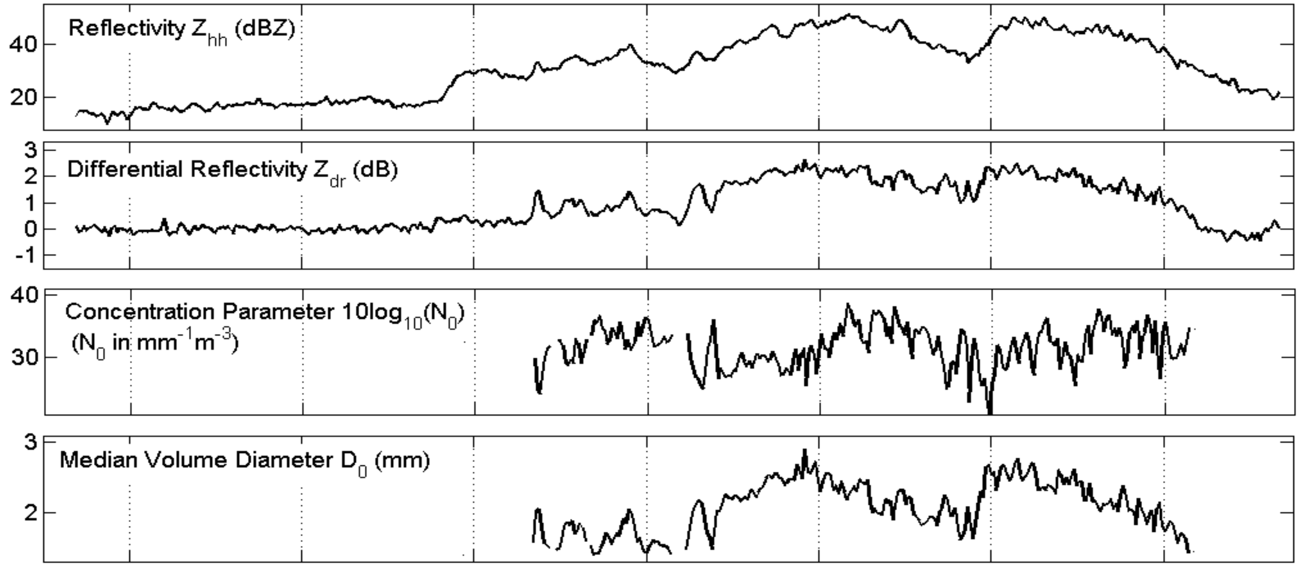
Polarimetry gives better estimate of N(D)
Because we can estimate two parameters of our model now.

Estimation of raindrop-size distribution

$$N(D) = N_0 \exp(-\Lambda D)$$

intercept ($m^{-3}mm^{-1}$) slope parameter (mm^{-1})

1. the differential reflectivity Z_{dr} depends only on the slope parameter Λ, so Λ can be directly estimated from Z_{dr}
2. once that the slope parameter is known, the concentration N_0 can be estimated in a second step from the reflectivity Z_{hh}



Data: IDRA (TU Delft), Jordi Figueras i Ventura

Observations and models revisited

$$z = \int_D D^6 N(D) dD \quad + \quad R = \frac{\pi}{6} \int_D D^3 v(D) N(D) dD \quad \Rightarrow \quad z = aR^b$$

$v(D)$: *terminal fall speed of raindrops*

$$\boldsymbol{v(D) = 9.65 - 10.3e^{-0.6D}}$$

$N(D)$: *dropsize distribution*

The model we use to describe that part of reality we need to transform radar observations into rainfall rate

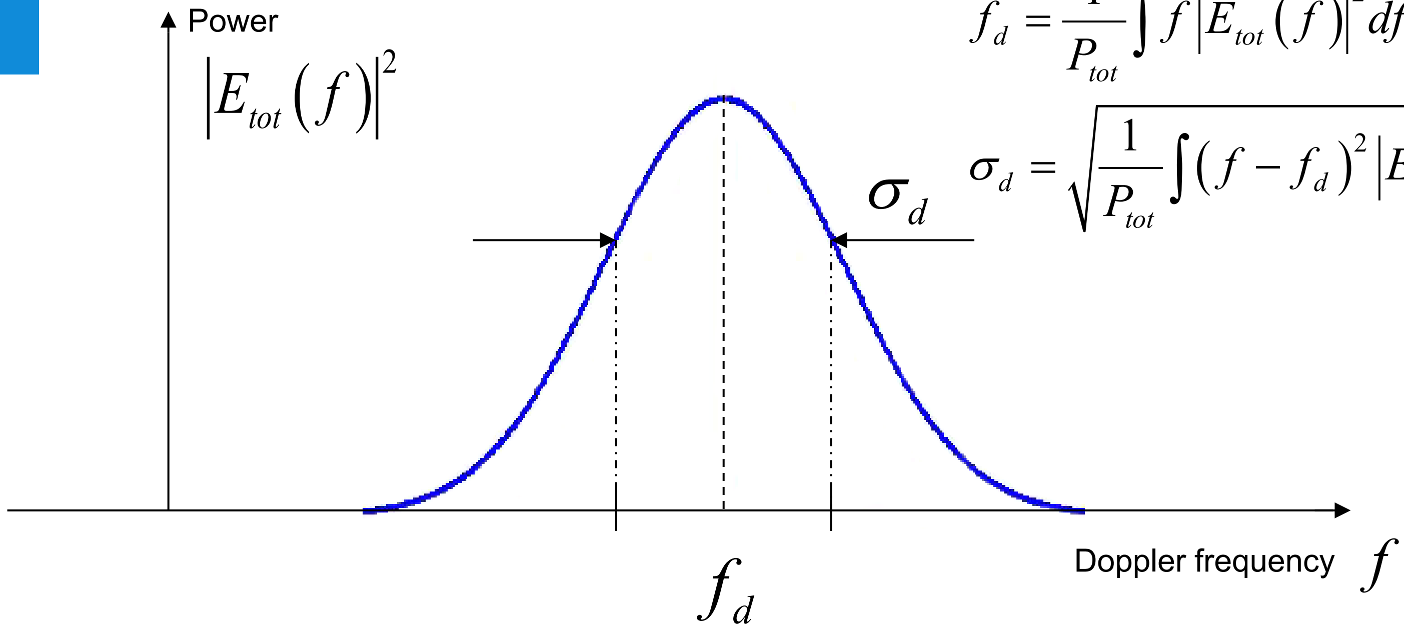
We used polarization to estimate the parameters of N(D).
We assumed a model for v(D).
Can we use v(D) to our advantage?

Recall the Doppler spectrum

$$P_{tot} = \int \left| E_{tot}(f) \right|^2 df$$ **Total power**

$$f_d = \frac{1}{P_{tot}} \int f \left| E_{tot}(f) \right|^2 df$$ **Mean doppler frequency**

$$\sigma_d = \sqrt{\frac{1}{P_{tot}} \int (f - f_d)^2 \left| E_{tot}(f) \right|^2 df}$$ **Doppler Width**



The Doppler frequency is related to the speed

When the radar looks upwards, the Doppler frequency gives the fall speed

$$f = \frac{2v(D)}{\lambda}\cos\theta;\ \theta = 0\ (to\ the\ vertical)$$

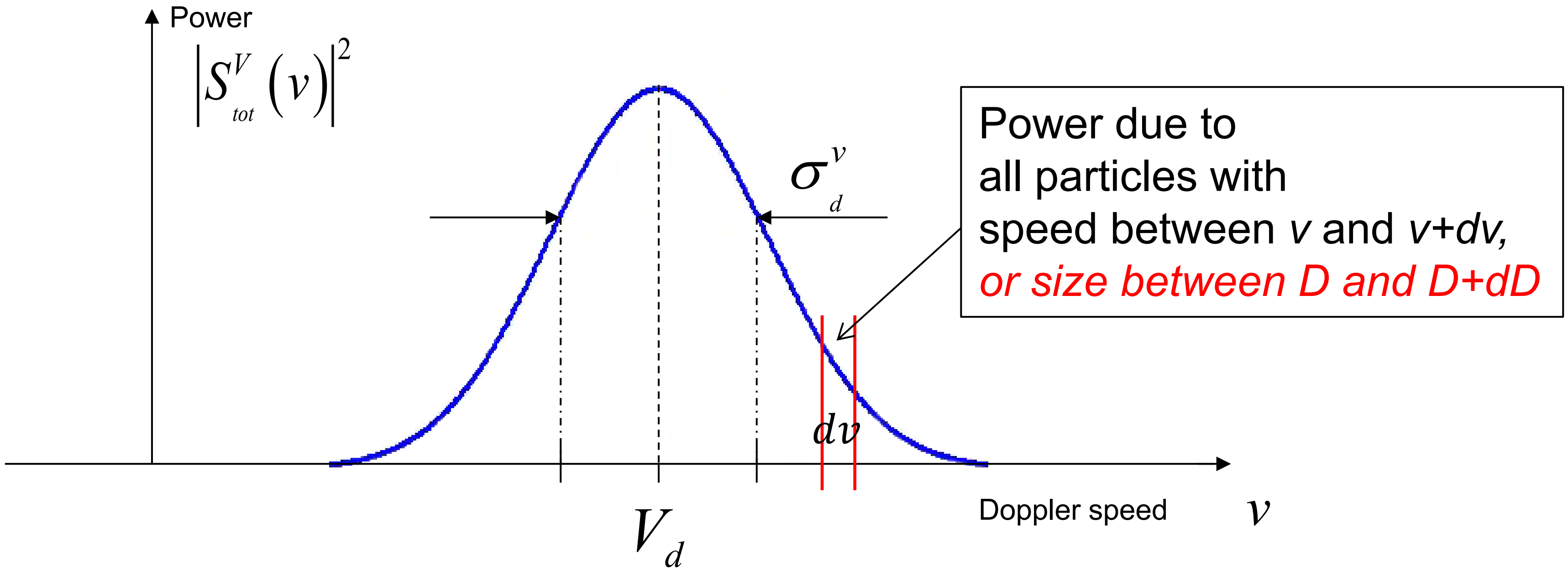
→ When we measure the Doppler spectrum,
we also measure dropsizes!

$$\left|E_{tot}(f)\,df\right| = \left|S_{tot}^{V}(v)\,dv\right| = \left|S_{tot}^{D}(D)\,dD\right|$$

$$\left|E_{tot}(f)\right| = \left|E_{tot}(f(v))\frac{dv}{df}\right| = S_{tot}^{V}(v)$$

$$\left|S_{tot}^{V}(v)\right| = \left|S_{tot}^{V}(v(D))\frac{dD}{dv}\right| = S_{tot}^{D}(D)$$

Recall the Doppler spectrum again



$$P_{tot} = \int |S_{tot}^{V}(v)|^2 dv$$

$$V_d = \frac{1}{P_{tot}} \int v |S_{tot}^{V}(v)|^2 dv$$

$$\sigma_d^v = \sqrt{\frac{1}{P_{tot}} \int (v - v_d)^2 |S_{tot}^{V}(v)| dv}$$

$$P_{tot} = \int |S_{tot}^{D}(D)|^2 dD$$

$$\langle D \rangle_d = \frac{1}{P_{tot}} \int D |S_{tot}^{D}(D)|^2 dD$$

$$\sigma_d^D = \sqrt{\frac{1}{P_{tot}} \int (D - D_d)^2 |S_{tot}^{D}(D)| dD}$$

The Doppler spectrum in terms of radar cross section

$$sZ_{HH}(v)dv = \underline{N(D\{v\})}\sigma_{HH}(D\{v\})\left|\frac{dD}{dv}\right|dv$$

So, if we measure the Doppler spectrum, we can retrieve the dropsize distribution

Complication:
- Doppler spectrum broadening by turbulence;
- Shifted by mean wind

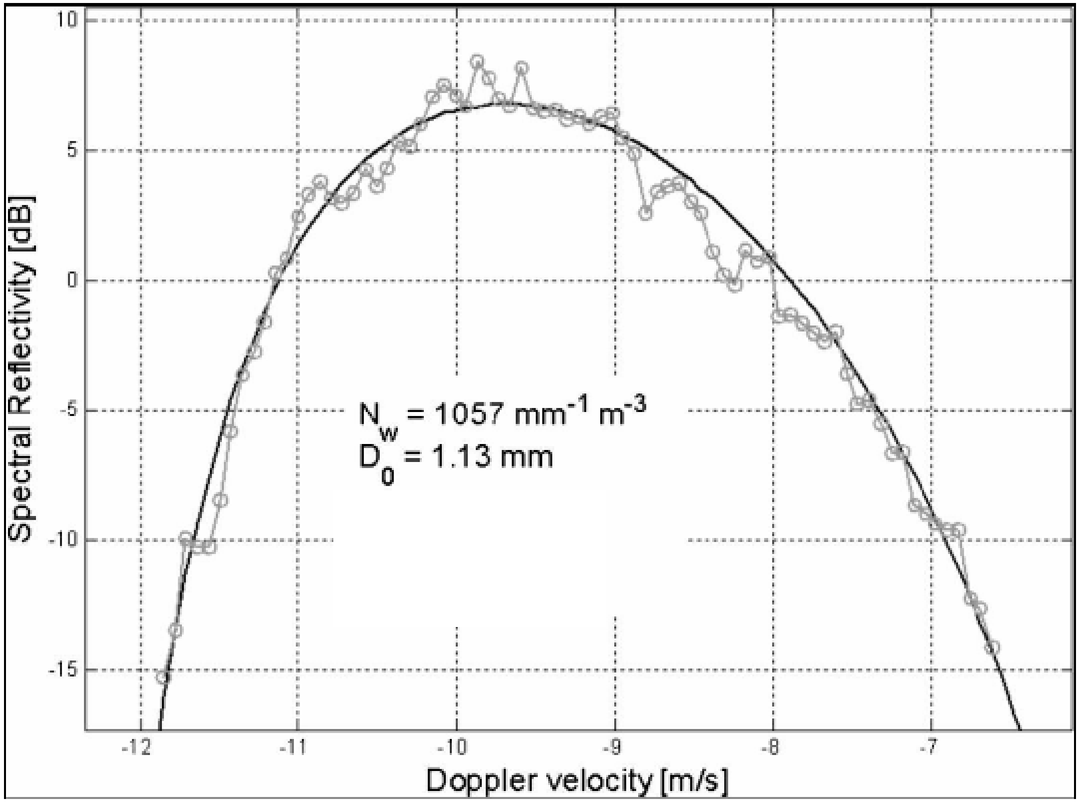
Procedure

Measure the Doppler spectrum
Compare the observation with the model
Change the model parameters (No, Do for instance)
Minimize the difference between the model and observation

Results: dropsize distribution

plus impact of errors due to turbulence and wind

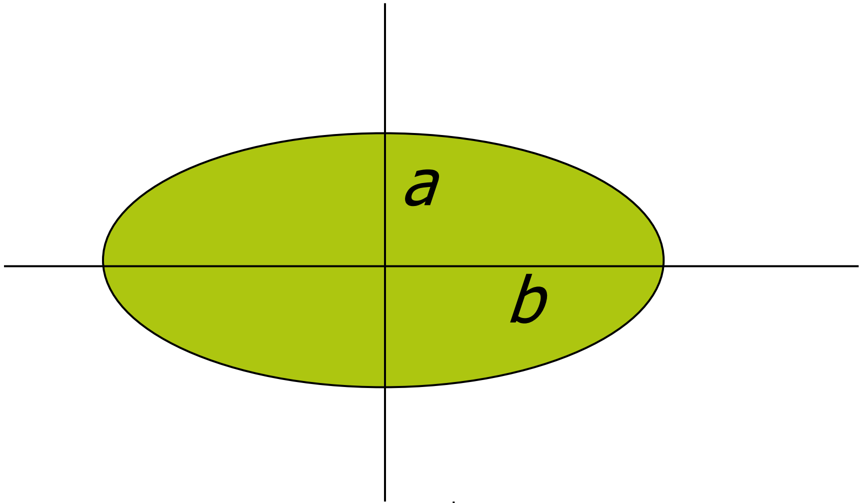
Example: measured Doppler spectrum plus curve fit



Courtesy Moisseev

Can we combine Doppler and polarization?

1- the fall speed depends on the particle size (Doppler)
2- the axial ratio depends on the particle size (polarization)



Model of axial ratio

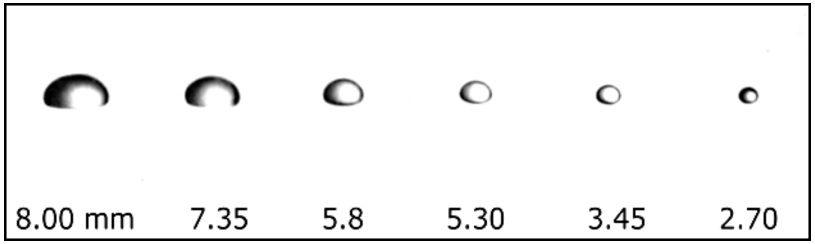
$$\frac{a}{b} = 1 - \beta D$$

$$\rightarrow \quad Z_{dr} = \frac{\int N(D)\sigma_{hh}(D,\beta)dD}{\int N(D)\sigma_{vv}(D,\beta)dD} = \frac{\int N^{v}(v)\sigma^{v}_{hh}(v,\beta)dv}{\int N^{v}(v)\sigma^{v}_{vv}(v,\beta)dv}$$

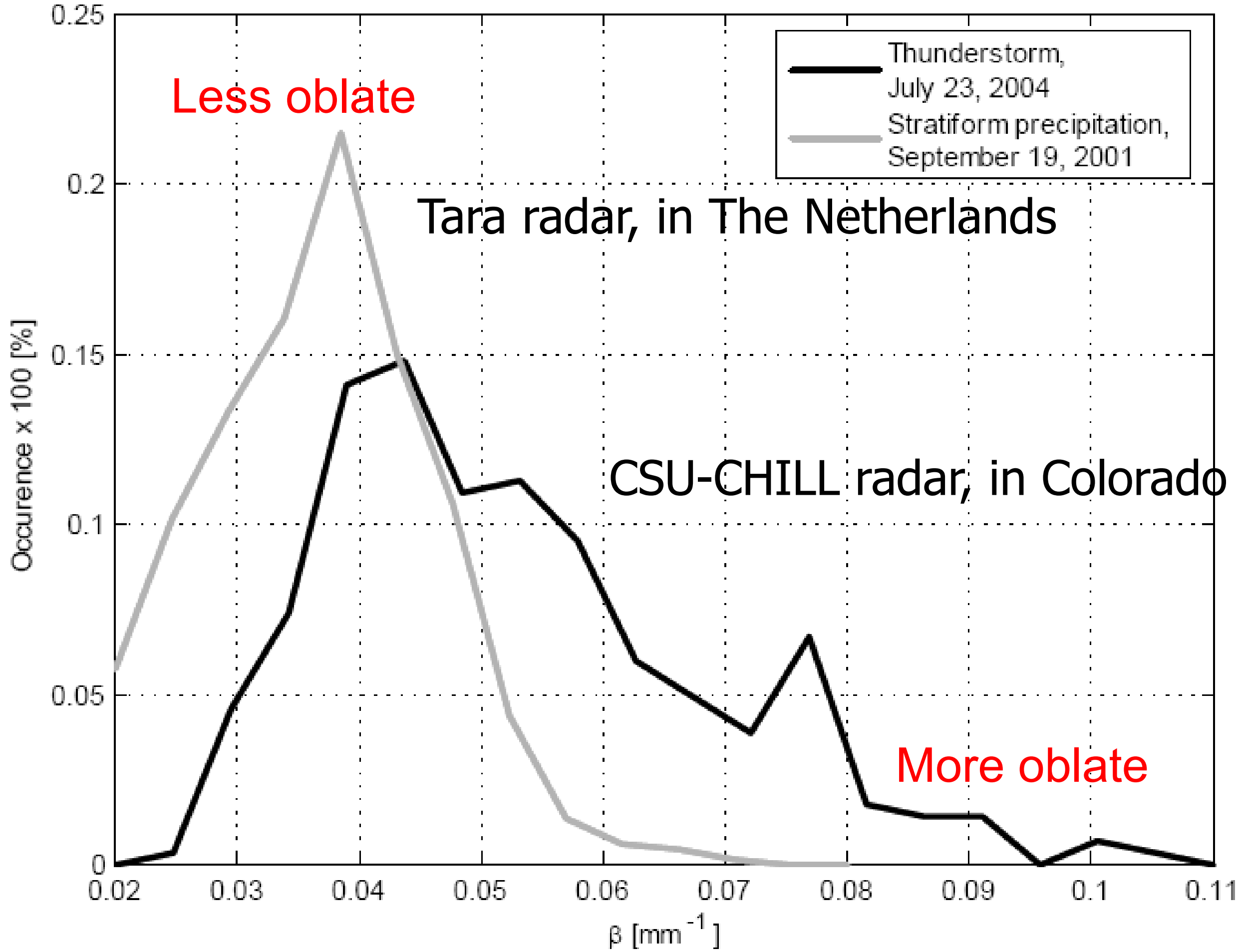
Combine the Doppler spectrum and the Zdr

Doppler spectrum → Dropsize distribution

Dropsize distribution + Zdr → $\frac{a}{b} = 1 - \beta \cdot D$

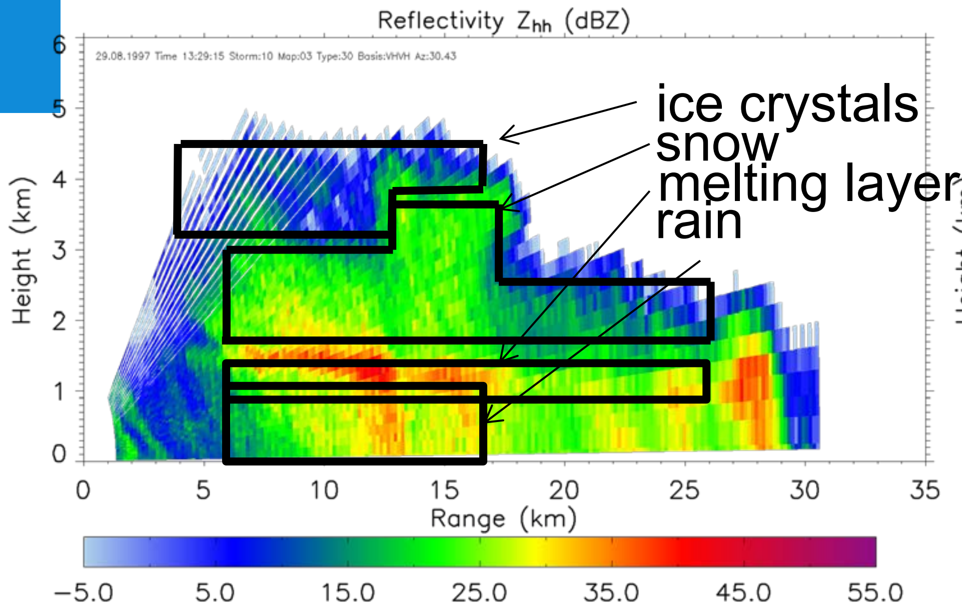


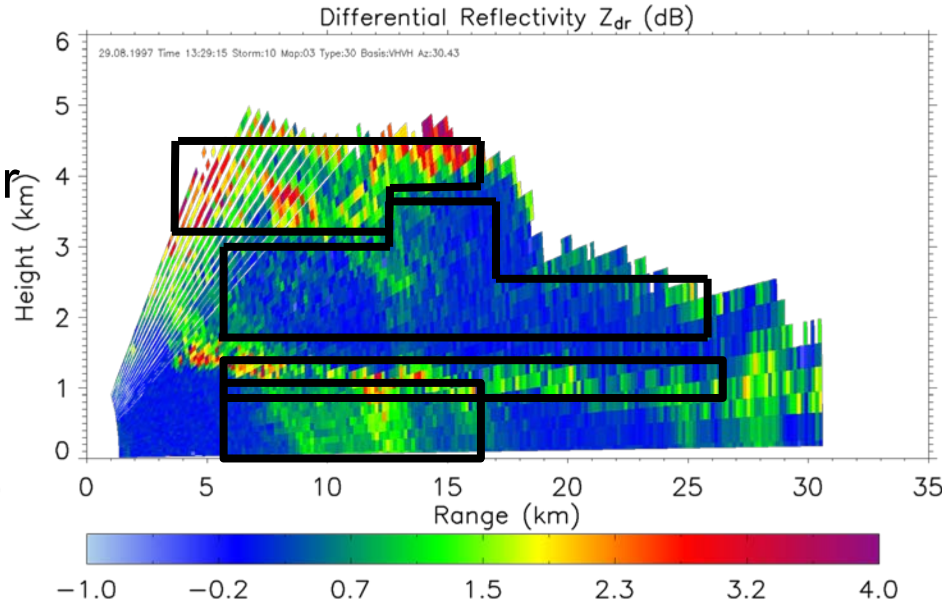
Example of retrieved drop shapes



Courtesy Moisseev

The Zdr can also be used for hydrometeor classification





Reflectivity

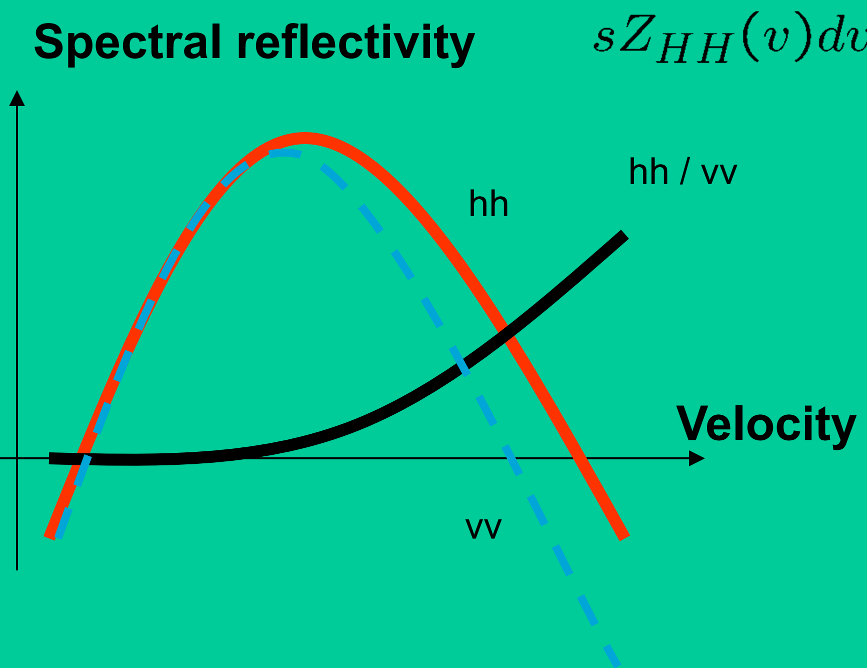
$$Z_{hh} = 10 \log C R^2 \overline{P}_{hh} (\text{dBZ})$$

Differential Reflectivity

$$Z_{dr} = 10 \log \frac{\overline{P}_{hh}}{\overline{P}_{vv}} (\text{dB})$$

Data: POLDIRAD (DLR, Oberpfaffenhofen, Germany), Prof. Madhu Chandra

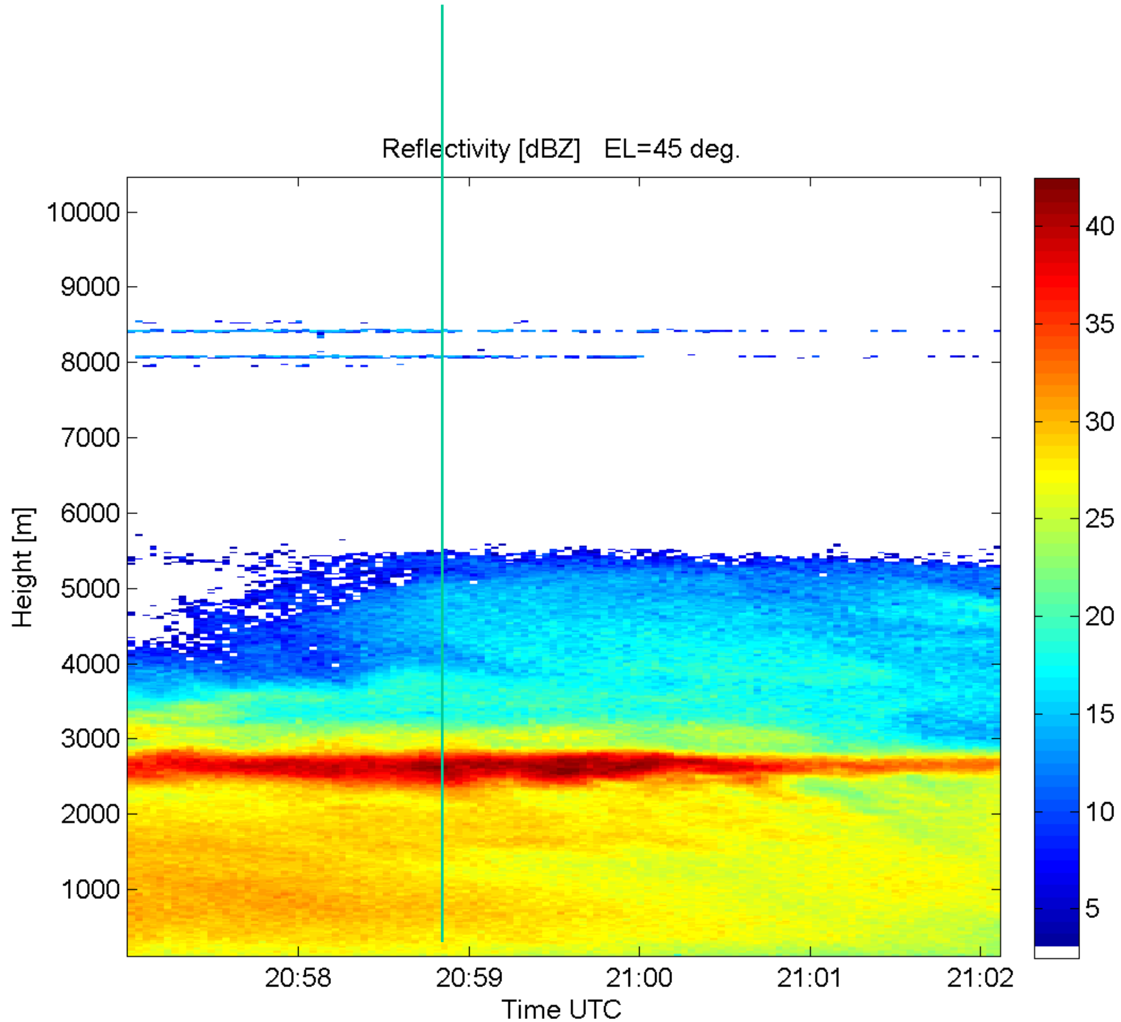
Spectral differential reflectivity



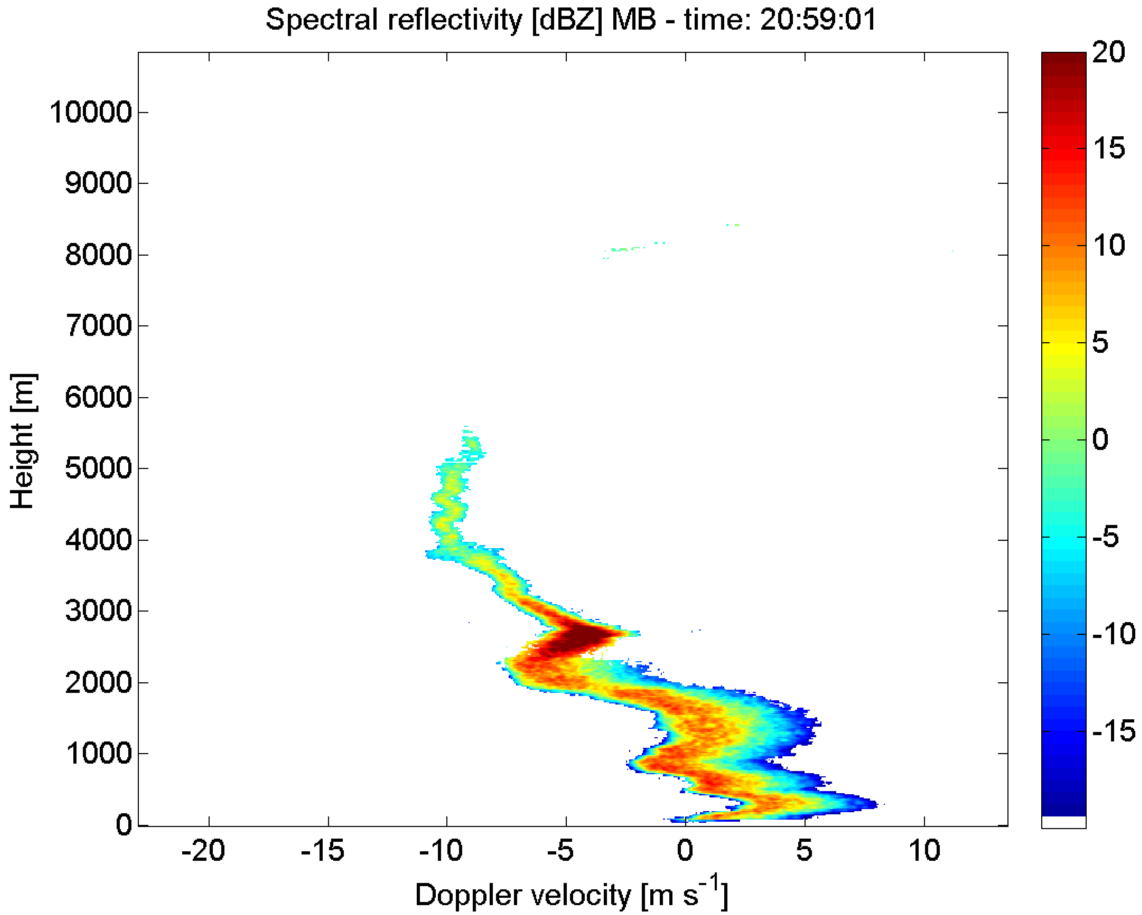
$$sZ_{HH}(v)dv = N(D\{v\})\sigma_{HH}(D\{v\})\left|\frac{dD}{dv}\right|dv$$

$$sZ_{DR}(v)dv = \frac{N(D\{v\})\sigma_{HH}(D\{v\})\left|\frac{dD}{dv}\right|dv}{N(D\{v\})\sigma_{VV}(D\{v\})\left|\frac{dD}{dv}\right|dv}$$

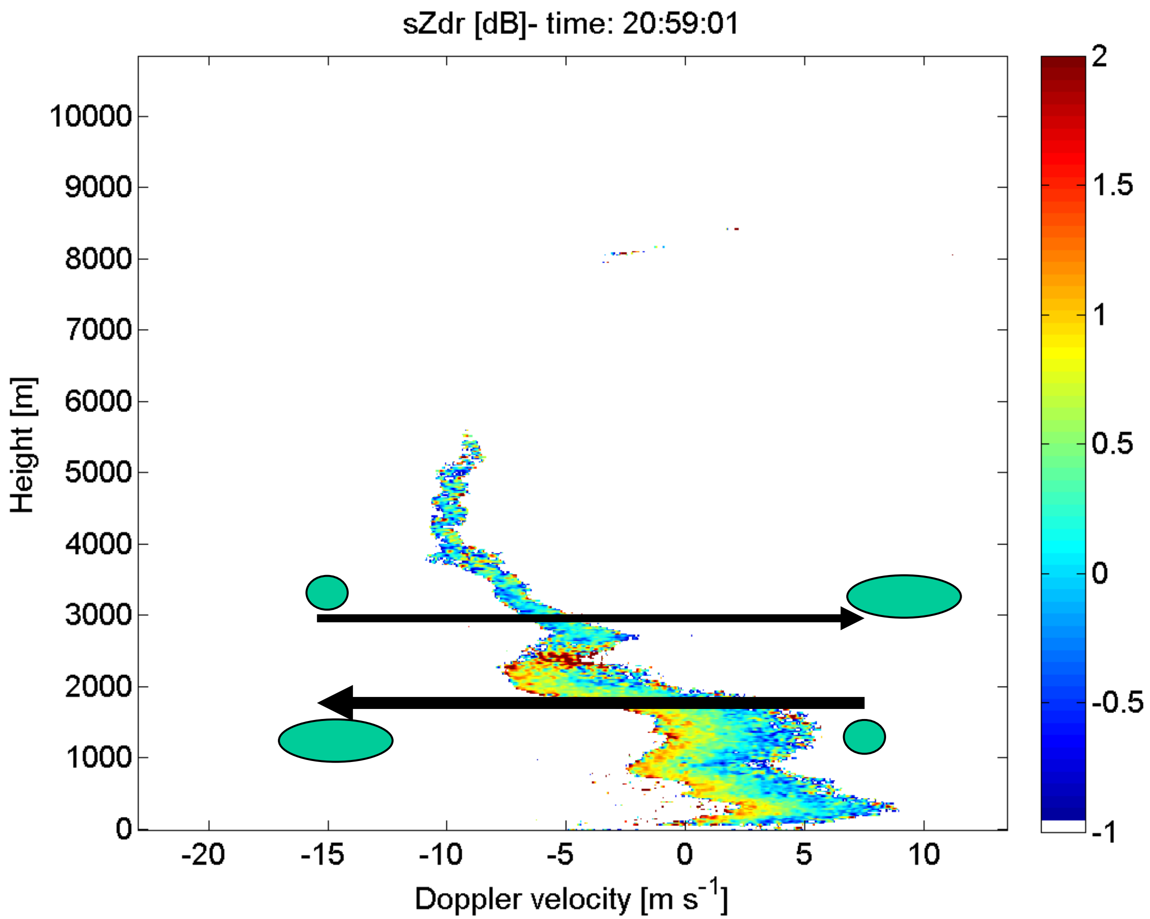
Time height reflectivity



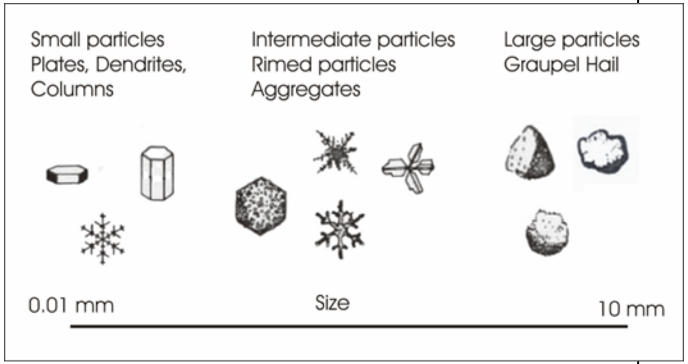
Spectrogram reflectivity

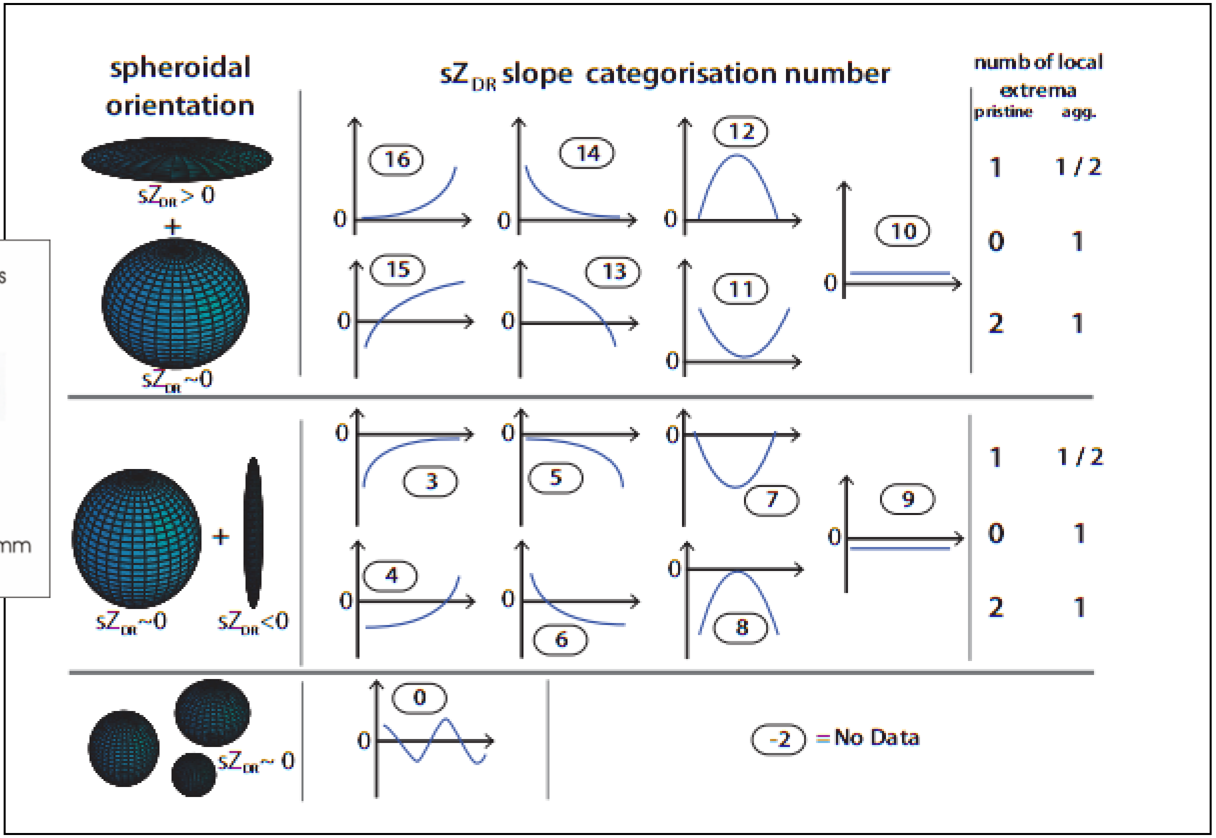


Spectrogram differential reflectivity

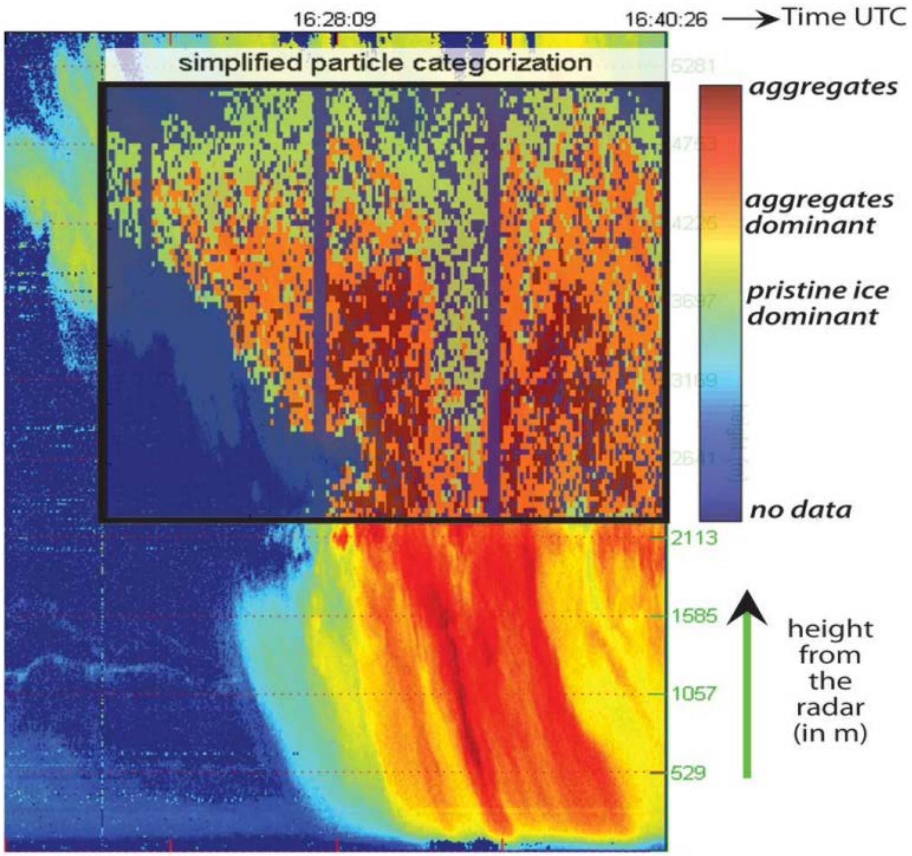


Ice crystal classification



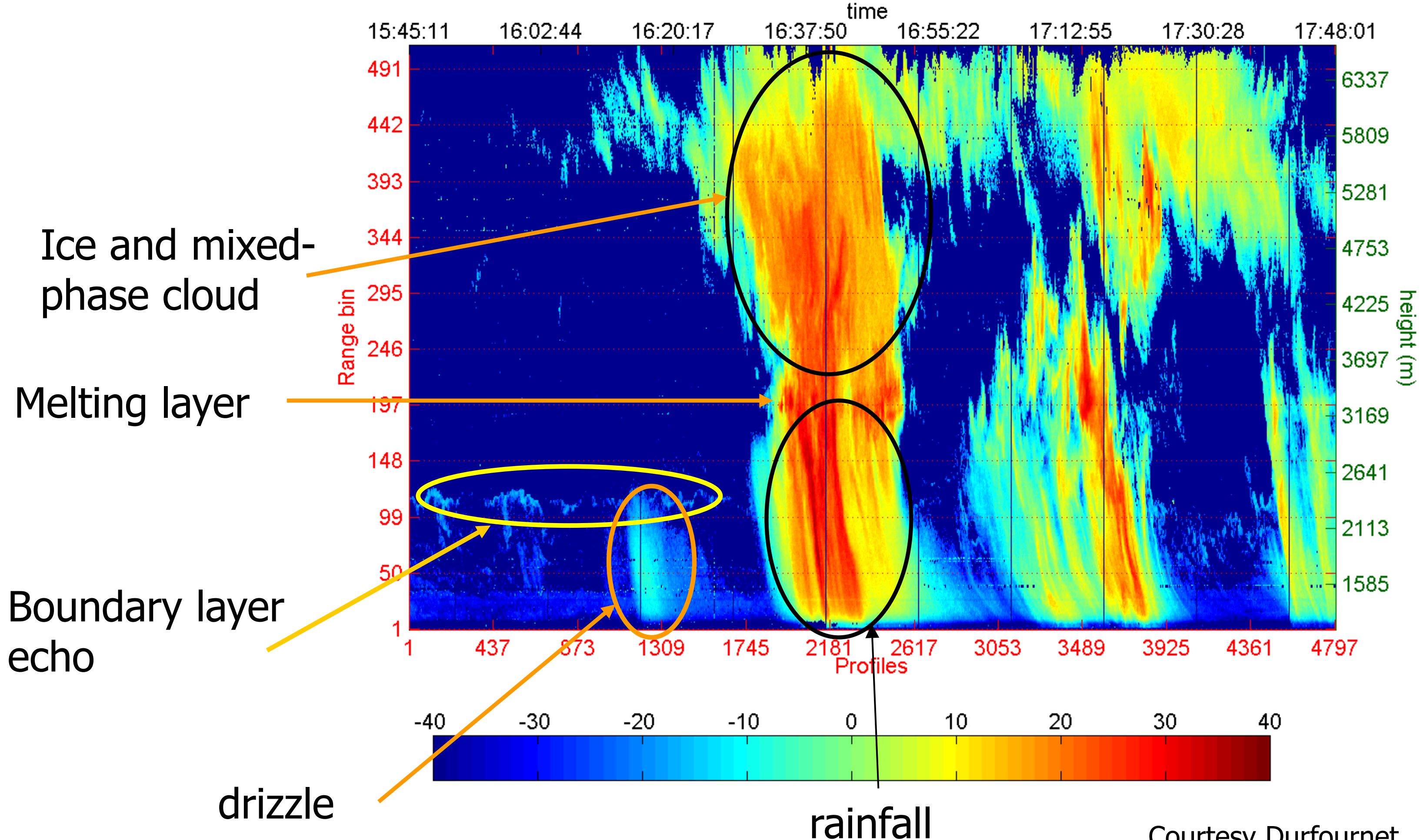


Spectral-polarimetric classification



Courtesy Durfournet

Observation plus model leads to a better understanding of rainfall formation



Courtesy Durfournet

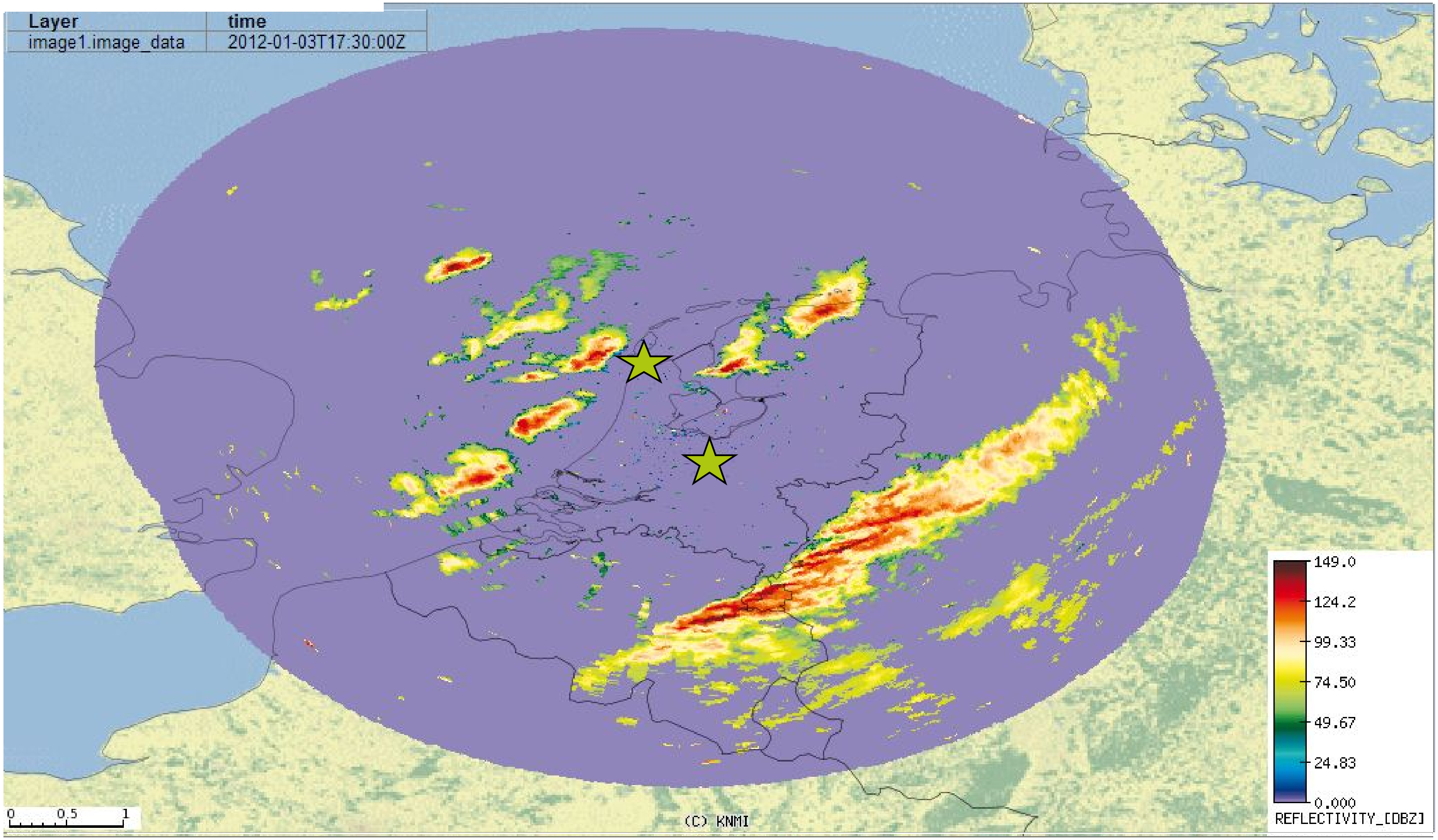
Short summary of remote sensing in this course

- Radar signals: behaviour, estimation of appropriate descriptors, accuracy
- The use of models for data interpretation
- Scattering by spheres
- Use scattering theory to define useful signal characteristics we need for the observations
- The inverse problem
- Application to Doppler-polarimetric weather radar

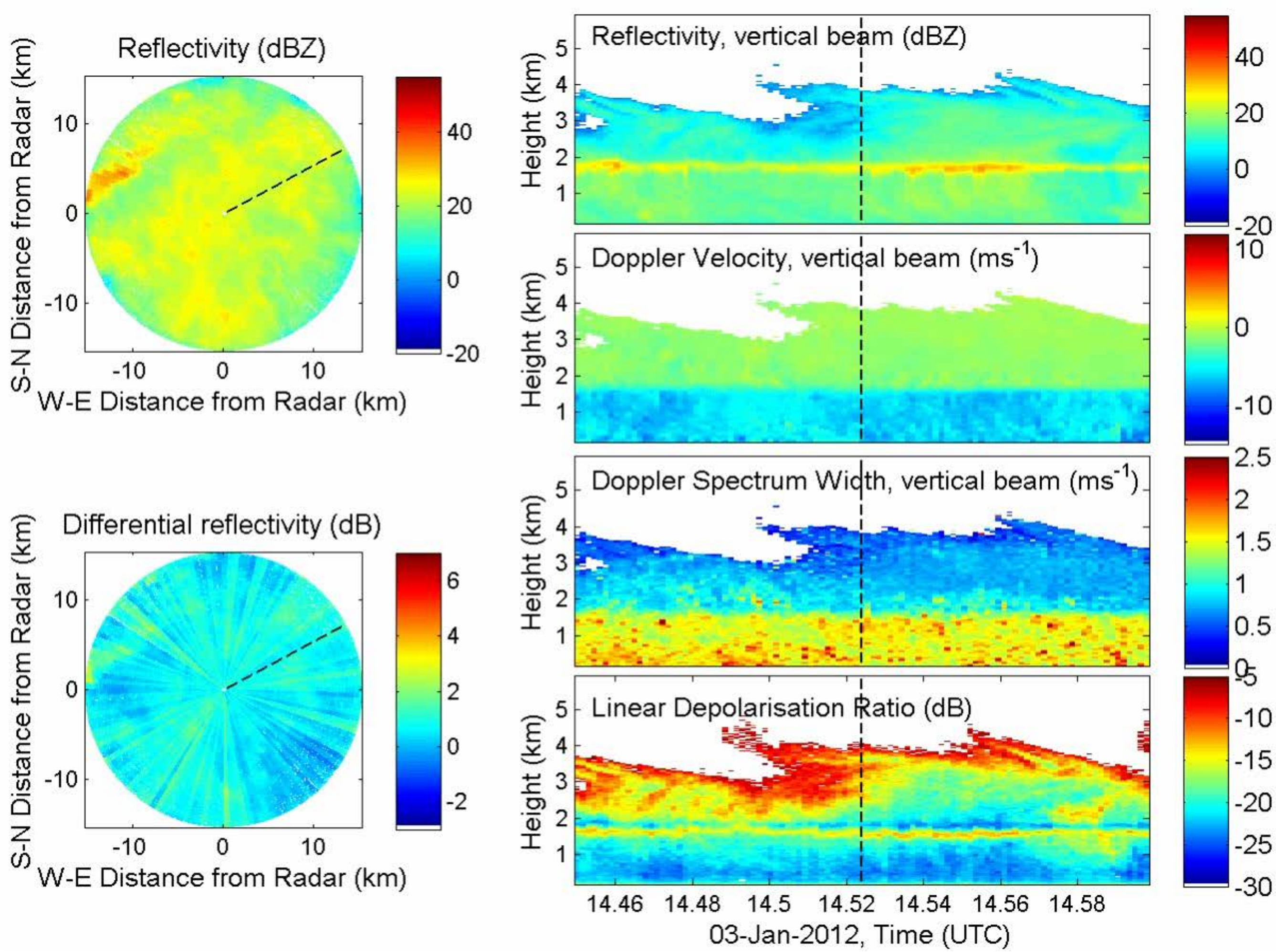
Composite KNMI C-band Radar

17:30 UTC

Radar position ★



Clouds and rainfall



Courtesy of Tobias Otto, Yann Dufournet, Christine Unal