Multi-Moment Formulation of the Autoconversion Process

Yangang Liu[1]

P. H. Daum[1], R. McGraw[1], M. Miller[1], and B. Geerts[2]

1-Brookhaven National Laboratory

2- University of Wyoming

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Autoconversion Parameterization

- **Autoconversion is the first step converting cloudwater to rainwater; autoconversion rate $P = P_0 T$ (P_0 is rate function & T is threshold function).**

- **Previous studies have been primarily on the autoconversion rate of the liquid water content (*mass autoconversion rate*).**

- **Aerosol indirect effects call for multi-moment schemes that further requires autoconversion rate of droplet concentration (*number autoconversion rate*).**

- **Few existing parameterizations for the number autoconversion rate are empirical at best, lacking clear physics as a result.**

Our focus has been deriving P_0 and T from first principles; this talk is a generalization of our previous work on mass autoconversion rate to multi-moment schemes, esp., droplet concentration.

General Formulation

$$Y = \alpha \int r^{\delta} n(r) dr$$

$$P_Y = P_{Y0} T_Y$$

$$P_{Y0} = \alpha \left(\frac{3}{4\pi\rho_w} \right)^{(6+\delta)/3} \kappa \beta_6^6 \beta_\delta^\delta N^{-\delta/3} L^{(6+\delta)/3}$$

$$T_Y = \gamma\left(\frac{6+q}{q}, x_{cq} \right) \gamma\left(\frac{\delta+q}{q}, x_{cq} \right)$$

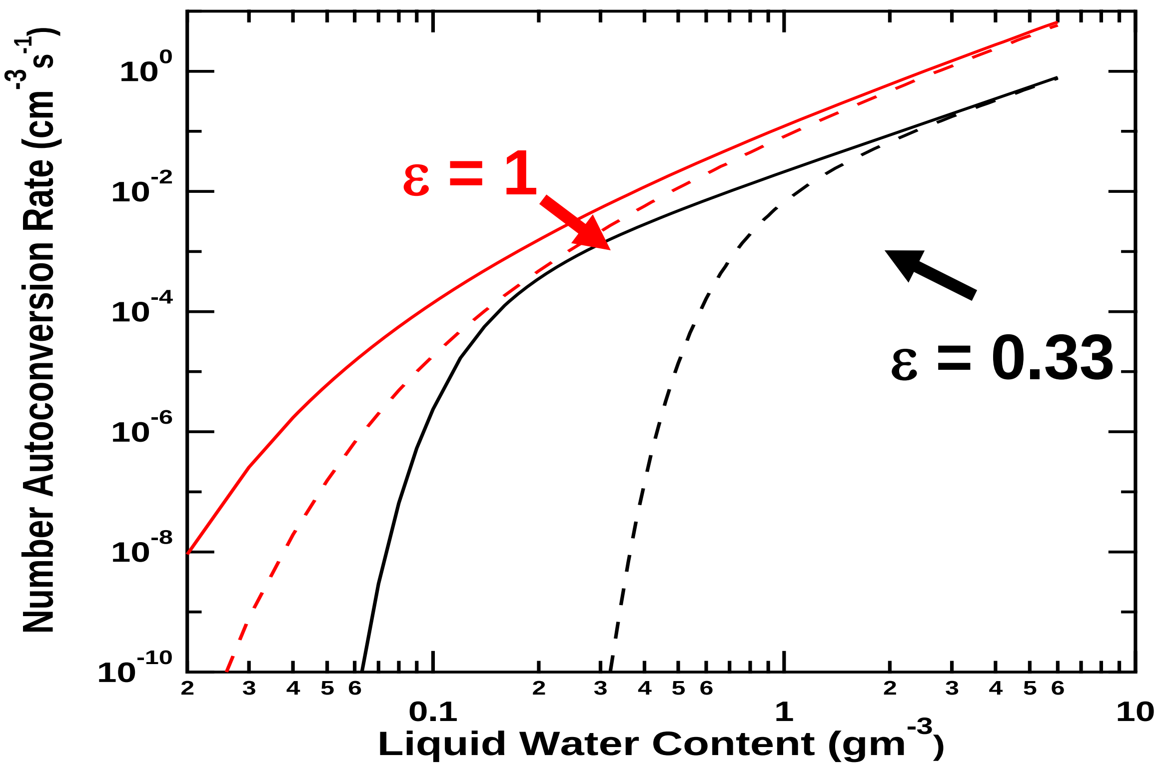
The parameters q and β are known functions of the relative dispersion (ε) of droplet size distribution; x_{cq} is a function of liquid water content (L) and droplet concentration (N).

Autoconversion rate of any moment is an analytical function of L, N and ε.

Two Special Cases

α	δ	Y
$\frac{4\pi\rho_w}{3}$	3	L
1	0	N

Number Autoconversion Rate



The solid and dashed lines represent those for N = 50 cm-3 and N = 500 cm-3, respectively.

The dependency exhibits two distinct regimes: threshold function dominated and rate function dominated. The smaller the relative dispersion ε, the more striking the distinction.

The Common Assumption

The assumption that has been commonly used to derive number autoconversion rate P_N from the mass autoconversion rate P_L is

$$P_N = \frac{3}{4\pi\rho_w r_*^3} P_L,$$

with a constant r_*

⬇

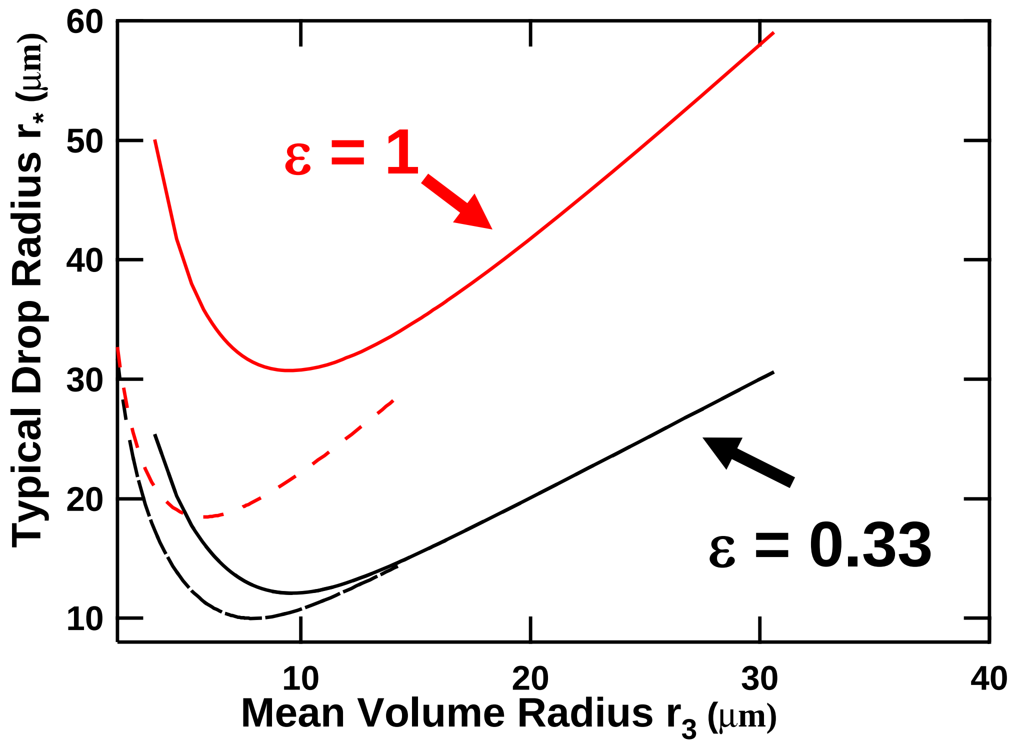
Is this assumption correct?

Examination of the Common Assumption

By combining the analytical expressions for the mass and number autoconversion rates, we obtain

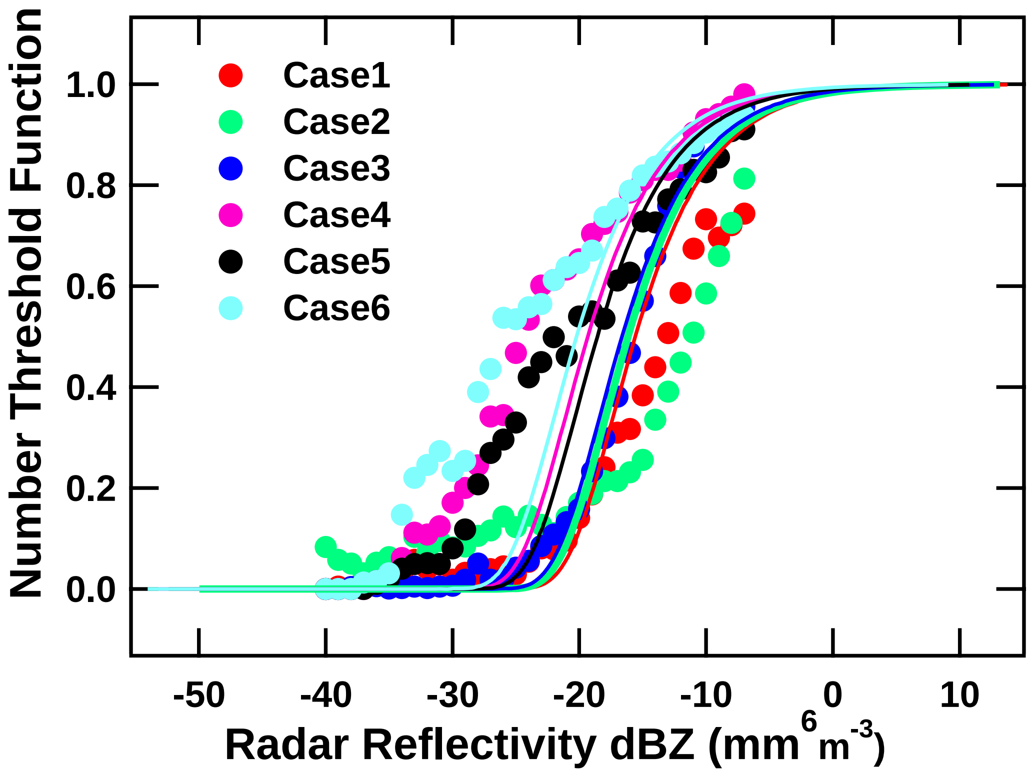
$$r_* = \left[\frac{\gamma\left(\frac{3+q}{q}, x_{cq}\right)}{\gamma\left(1, x_{cq}\right)} \right]^{1/3} r_3$$

r_3 = mean volume radius



This figure shows that the common assumption of a constant r_* is incorrect. The detail also depends on the droplet concentration and relative dispersion. The solid and dashed lines represent those for N = 50 cm^{-3} and N = 500 cm^{-3}, respectively.

Observational Validation



This figure compares the theoretical threshold function (solid curves) to the observational results (dots). Evidently, the theoretical expression compares favorably with observations. The discrepancies may result from different values of relative dispersion.

Summary

- **Closed form multi-moment formulation is established for the autoconversion process.**

- **The general formulation includes mass and number autoconversion rates as special cases.**

- **Autoconversion rate depends generally on liquid water content, droplet concentration and relative dispersion.**

- **The formulation also has other applications such as cloud radar reflectivity.**

(Please see our poster for detail)