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RSS algebra and noise

For one shadowband cycle RSS produces four 1040 long columns (*C1,…, C4*) of counts that were obtained at the same exposure *E* (see Table 1 for notations). The cosine response correction *cdf* for diffuse irradiance is obtained directly from a lookup table and cosine response correction *cdr* for direct irradiance is calculated from two lookup tables using solar zenith and azimuth angles.

Table 1. Notation and units

Sun unblocked	*C1*	cts
Average ±9° correction	*C2*	cts
Sun blocked	*C3*	cts
Dark (closed shutter)	*C4*	cts
Exposure (100=1sec)	*E*	100=1sec
Direct cosine correction	*cdr*	No units
Diffuse cosine correction	*cdf*	No units
Ad hoc coefficient	*A=(1/cdr-1/cdf)*	No units
Solar zenith angle	*sza*	rad
Variance of C1	*V1*	cts^2
Variance of C2	*V2*	cts^2
Variance of C3	*V3*	cts^2
Variance of C4	*V4*	cts^2
Counts-to-Electrons coefficient	*k*	cts/e
Counts offset	*C0*	cts
Readout noise variance	*R*	cts^2
Standard Deviation (as fraction)	*S*	No units
Responsivity	*Resp*	cps/(W/m^2/nm)

The direct, diffuse and total horizontal components are calculated from *C1,…, C4, cdf* and *cdr* from formulas given in Table 2. Also one can calculate variance of direct, diffuse and total horizontal using formulas in Tables 2 and 3. The standard deviation *S* expressed as a fraction of signal is given in last column of Table 2. We prefer using *S* as it is independent of radiometric calibration and both direct and diffuse are independent of respective cosine corrections.

Table 2. Direct, diffuse, total and variance formulas

	Counts formula	Variance formula	Std.Dev. Fraction
Direct horizontal cosine corrected	*C=(C2-C3)/cdr*	*V=(V2+V3)/cdr^2*	*S=sqrt(V)/C* (cdr cancels)
Direct normal cosine corrected	*C=(C2-C3)/cdr/cos(sza)*	*V=(V2+V3)/cdr^2/(cos(sza))^2*	*S=sqrt(V)/C* (*cdr* and *cos(sza)* cancel)
Diffuse horizontal cosine corrected	*C=(C1-C2+C3-C4)/cdf*	*V=(V1+V2+V3+V4)/cdf^2*	*S=sqrt(V)/C* (*cdf* cancels)
Total horizontal cosine corrected	*C= (C2-C3)/cdr+ (C1-C2+C3-C4)/cdf = (C2-C3)A+(C1-C4)/cdf*	*V=(V2+V3)A^2+(V1+V4)/cdf^2*	*S=sqrt(V)/C* (*cdf* and *cdr* do not cancel)

Table 3. Variance formulas

	Unblocked	Correction	Blocked	Dark
Counts	*C1*	*C2*	*C3*	*C4*
Variance formula	*V1=k(C1-C0)+R*	*V2=0.5k(C2-C0)+0.5R*	*V3=k(C3-C0)+R*	*V4=k(C4-C0)+R*

Counts normalized to 1 sec are obtained from substitution:

$$C=C/(E/100)$$

and the irradiances in W/m^2/nm are obtained after dividing by the responsivity:

$$C=C/Resp$$

The noise parameters *k, C0* and *R* for RSS105 are given in Table 4. Their derivation is presented in the Appendix.

Table 4. Noise parameters of RSS105

Counts-to-Electrons coefficient	*k*	*0.1458cts/e*
Counts offset	*C0*	*168cts*
Readout noise variance	*R*	*11.04 cts^2*

Notes on calculations

In direct, diffuse and total counts *C* produced by the formulas from Table 2 are forced to be nonnegative (if $C<0$ then $C=0$).

In variance formulas (Table 3) we force $Ci-C0$ (for $i=1,\ldots,4$) to be nonnegative (if $Ci-C0<0$ then $Ci-C0=0$).

In standard deviation formula (Table 2) we set $S=0$ if the denominator $C=0$ and if $S>1$ then we set $S=1$.

Derivation of total horizontal only

In cases when the band is not shading correctly only total horizontal component can be estimated. Then C2 and C3 are not valid. Only C1 and C4 are valid measurements, however at some instances C1 might not be valid if the band happened to be above horizon and this would be particularly true if the band was blocking the sun during the C1 measurement.

Table 5. Total horizontal only formulas

Total horizontal counts	$C= (C1–C4)/cdf$
Variance of total horizontal	$V=(V1+V4)/cdf$^2
Standard deviation	$S=sqrt(V)/C$
Normalized total horizontal	$C=C/(E/100)$
Total horizontal irradiance	$C=C/Resp$
Direct normal	-999
Diffuse horizontal	-999

One cannot detect from C1, C2, and C3 values whether shadowbanding was correct. However by evaluating daily profiles, the defects in shadowbanding sometimes can be deduced. The formulas from Table 5 should be used when we know a priori that shadowbanding was invalid. For instance this would be the case for most days in September 2004.

Linearization

Some instruments will exhibit nonlinear dependence between counts and irradiance. Also the reported exposure may require correction.

To linearize counts we use the following parameters:

Table 6: Counts linearizer parameters

Linearize if YesOrNo=1	*YesOrNo*
1st parameter	*k0*
2nd parameter	*k1*
3rd parameter	*k2*
Counts offset from Table 4	*C0*

To perform linearization we do the following substitutions:

C1=C1-C0
C2=C2-C0
C3=C3-C0
C4=C4-C0

C1= ***CountsCorrected****(C1,YesOrNo,k0,k1,k2) + C0*
C2= ***CountsCorrected****(C2,YesOrNo,k0,k1,k2) + C0*
C3= ***CountsCorrected****(C3,YesOrNo,k0,k1,k2) + C0*
C4= ***CountsCorrected****(C4,YesOrNo,k0,k1,k2) + C0*

Then new corrected *C1,C2, C3* and *C4* are used in all calculations to get variances and *dir, dif* and *tot* irradiances as outlined in Tables 3 and 4.

The correction of exposure is obtained with the following function

E= ***ExposureCorrected****(E,YesOrNo,e1,a1,b1,e2,a2,b2)*

Where function ***ExposureCorrected()*** is defined in the Appendix and its parameters in the following table

Table 7: Exposure linearizer parameters

Linearize if YesOrNo=1	*YesOrNo*
First breakpoint	*e1*
1st parameter for *[0,e1]* interval	*a1*
2nd parameter for *[0,e1]* interval	*b1*
Secondbreakpoint	*e2*
1st parameter for *[e1,e2]* interval	*a2*
2nd parameter for *[e1,e2]* interval	*b2*

Note*:* function ***ExposureCorrected()*** is identity for *E>e2.*

The counts normalized to 1 sec are obtained from substitution:

C=C/(E/100)

Note: It appears that to describe the current nonlinearity in RSS105 not all parameters of linearizers will be used.

Appendix: linearizer functions

```
//This func returns corrected counts
function  CountsCorrected(cts, YesOrNo, k0,k1,k2 )   //Example of parameters for rss104 would be  {1,0,-12.5e-07,0}
variable cts, YesOrNo, k0,k1,k2

variable r

r=cts

if(YesOrNo==1)
	if(cts>0)				// This is so because cts^k0 can get flaky for cts<0 and even for cts=0

		r=cts*cts^k0*exp((k1+k2*cts)*cts)

	endif

endif

return r

end

//This function returns corrected exposure in three intervals (0,e1) (e1,e2) (e2,inf)
function ExposureCorrected(expo,YesOrNo,  e1, a1,b1, e2, a2,b2)			// Should be e1<e2 and expo>0
variable expo,YesOrNo,  e1, a1,b1, e2, a2,b2				// expo, e1 and e2 are in the same units
							// Let us always use  1sec=100 units  (in SI it is centiseconds [cs])
			// If only one breakpoint is sufficient then e1 should be negative and  a1,b1 are not used
			//Example of parameters for rss104 would be  {1,-1,0,0, 100,0.9955,045}
variable r

r=expo
if(YesOrNo==1)
	if(expo<=e1)
		r=r*(a1+b1/expo)
	else
		if(expo<=e2)
			r=r*(a2+b2/expo)
		endif
	endif
endif

return r

end
```

Appendix: derivation of noise parameters

To obtain dark counts offset *C0* we measure counts with closed shutter for different exposure values (see Figure 1). *C0* is obtained by linear extrapolation as value of counts for exposure *E=0*. *C0* may change when the offset value on the CCD board is changed.

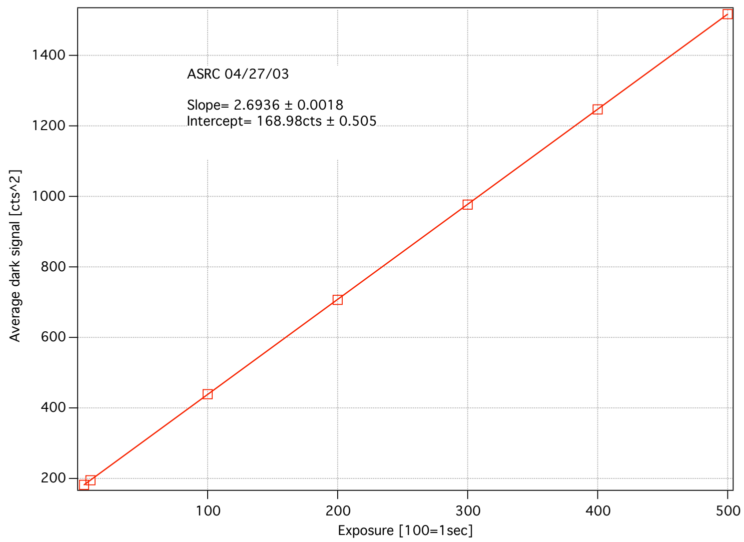

Figure 1. Average dark versus exposure.

To obtain readout noise variance *R* we measure variance at each pixel of net signal with closed shutter (see Figure 2). Then the variance is averaged over all pixels. The intercept of the straight line (averaged variance vs. exposure) is equal to *2R*. The readout noise depends on CCD and A/D circuit and in particular on implementation of the dual slope integration. Also *R* is slightly temperature dependent.

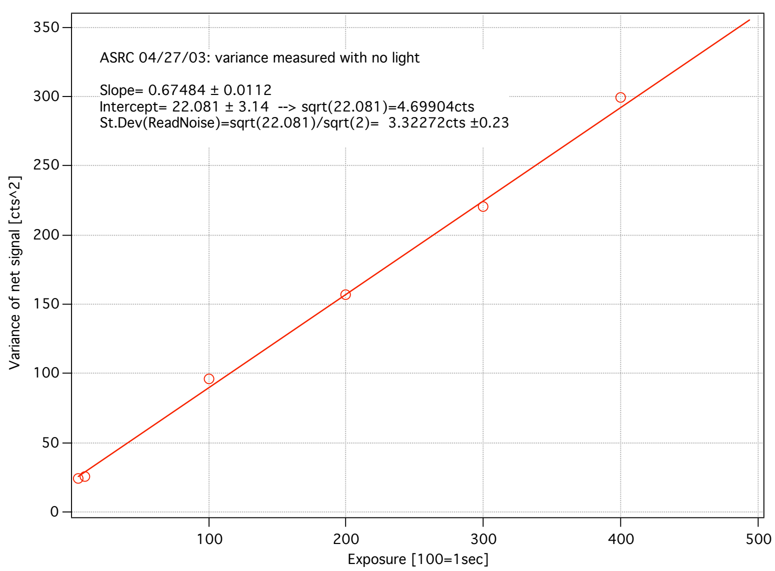

Figure 2. Variance of net signal versus exposure.

To obtain the count-to-electron coefficient *k* we measure variance at each pixel of net signal with light on for several exposures. Then we plot variance versus *Net+2Dark* counts. The slope of the straight line should be equal to k and the intercept should be equal to $2R$. This method is not suitable for precise estimate of R. however, it provides good estimate of k. We had only *4%* standard deviation in k value among five measurements with five different exposures. The count-to-electron coefficient k depends on gain of A/D electronics.

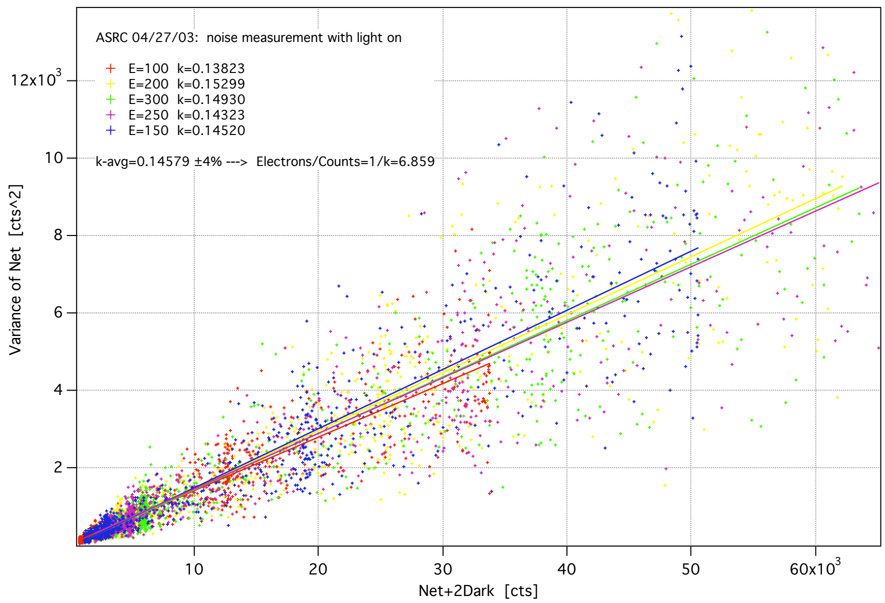

Figure 3. Variance of net signal for each pixel versus counts (*Net +2Dark*) at that pixel.

Note: The readout noise is often quoted as standard deviation in electrons. Thus $\sigma=sqrt(R)/k=22.7e$. This is still about 4 times more than the theoretical limit quoted by EEV the CCD's manufacturer.