The dependence of $r$ for $Q_s$ decreases as $r$ varies. For $r < 1$ and $m = 1.40$, $Q_s$ approaches the value $2$ when $r >> \lambda$.

The wavelength dependence of $\sigma_p$ is characterized by the Ångström exponent $\alpha$, defined by $\sigma_p \propto \lambda^{-\alpha}$. The approximation is quite accurate.

A simple expression for $Q_s$ would simplify calculation of many aerosol optical properties and would provide insight into the dependence of these properties on wavelength, index of refraction, RH, and the characteristic size of the aerosol.

**Approximation of $Q_{se}$**

Much of the variability in $Q_{se}$ can be eliminated by plotting it against the quantity $z = 2(2\pi r/\lambda)(m - 1)$. For $\lambda = 0.50$ $\mu$m and $m = 1.40$, $z = 10(\pi r/\lambda)$.

Wiggles are unimportant, as they are smoothed out by the spread in the size distribution.

In many situations the majority of the scattering is from the range $0.1 < r/\lambda < 0.7$ and $0 < z < 7$, over which $Q_{se}$ can be accurately approximated by a gamma function:

$$Q_{se} = 0.85 + 0.15 \exp(-z).$$

For a size distribution which is a gamma function with effective radius $r_{eff}$ and effective variance $\sigma_{eff}^2$, this approximation yields an analytic expression for the scattering cross section $\sigma_p = 0(\exp(\alpha z) - 1)$, where $\alpha$ is the Ångström exponent, and for the Ångström exponent $\alpha$,

$$\alpha = \frac{2\pi r_{eff}}{\lambda}(1 - \frac{\lambda}{2\pi r_{eff}}).$$

explicitly showing the dependences of these quantities on properties of the size distribution (i.e., $r_{eff}$ and $\sigma_{eff}$). The dependence of $\alpha$ on RH is not simple, as $\sigma_{eff}$ increases but $m$ decreases with increasing RH.

**Comparison of $\sigma_{eff}$**

The single mode fit provides a good approximation (within 10-20%) for $0.05 \mu$m < $r_{eff}$ < $0.7 \mu$m, and the double mode fit for $0.1 \mu$m < $r_{eff}$ < $1.2 \mu$m.

Most measurements of $d$ yield values between $0$ and $2$. The approximation is quite accurate.

**Conclusions**

- $Q_{se}$ depends mostly on $z = 4\pi r/\lambda(m - 1)$.
- $Q_{se}$ can be accurately approximated by a sum of gamma functions over the range of interest.
- Analytic results for $\sigma_p$ and $d$ can be obtained for size distributions which are gamma functions or sums of gamma functions.
- This method can also be applied to lognormals.