DETERMINATION OF RADIAL MOMENTS OF AN AEROSOL SIZE DISTRIBUTION FROM MEASUREMENTS OF LIGHT TRANSMITTANCE AND SCATTERING

Ernie R. Lewis and Stephen E. Schwartz

elewis@bnl.gov ses@bnl.gov

Brookhaven National Laboratory, Upton, NY 11933

BROOKHAVEN NATIONAL LABORATORY

MOTIVATION

Determination of the moments of an aerosol size distribution from measurements of light transmittance and scattering at various angles and wavelengths may provide a robust means to obtain information about the distribution.

This information could be used to calculate aerosol quantities of interest and aerosol radiative forcing, and is well suited to use in models of aerosol chemistry and transport.

RADIAL MOMENTS OF A SIZE DISTRIBUTION

$$\mu_i = \int_0^\infty r^i \frac{dn}{dr} dr$$

- i can be >0, <0, or fractional
- μ_i may or may not exist

The lowest several integral moments contain important information about the size distribution, and can be used to determine many quantities of interest. They are easy to use in models (such as GChM) and require less overhead than bins.

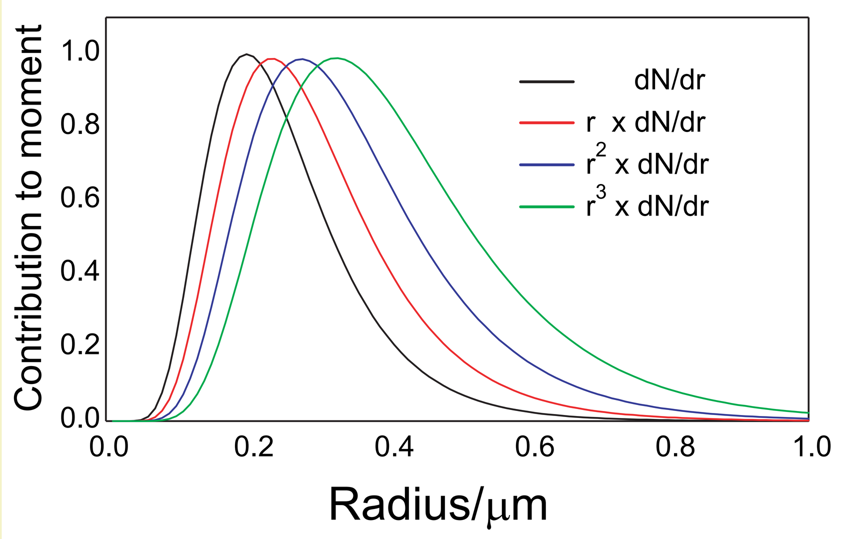
Extensive quantities related to moments:

- Number concentration $N_{tot} = \mu_0$
- Surface area $A = 4\pi\mu_2$
- Volume $V = (4\pi/3)\mu_3$

Intensive quantities related to moments:

- Effective radius $r_{eff} = \mu_3/\mu_2$
- Effective variance $v_{eff} = \mu_4\mu_2/(\mu_3)^2-1$

Larger particles contribute more to higher moments:

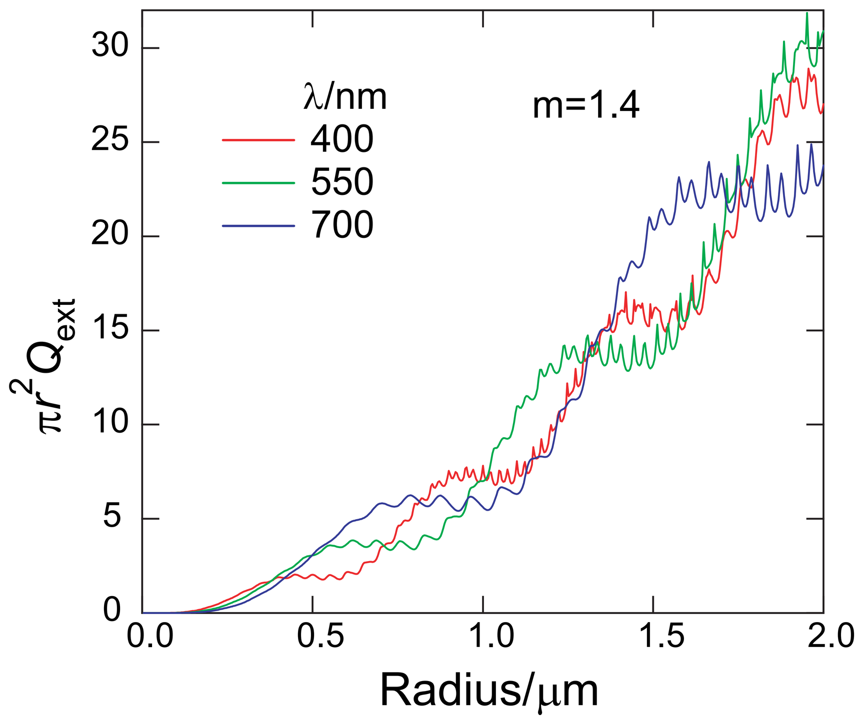


MEASURABLE OPTICAL QUANTITIES

Information about the size distribution and composition of an aerosol can be obtained from measurements, at different wavelengths, of light *transmittance* and of *scattering* at different angles.

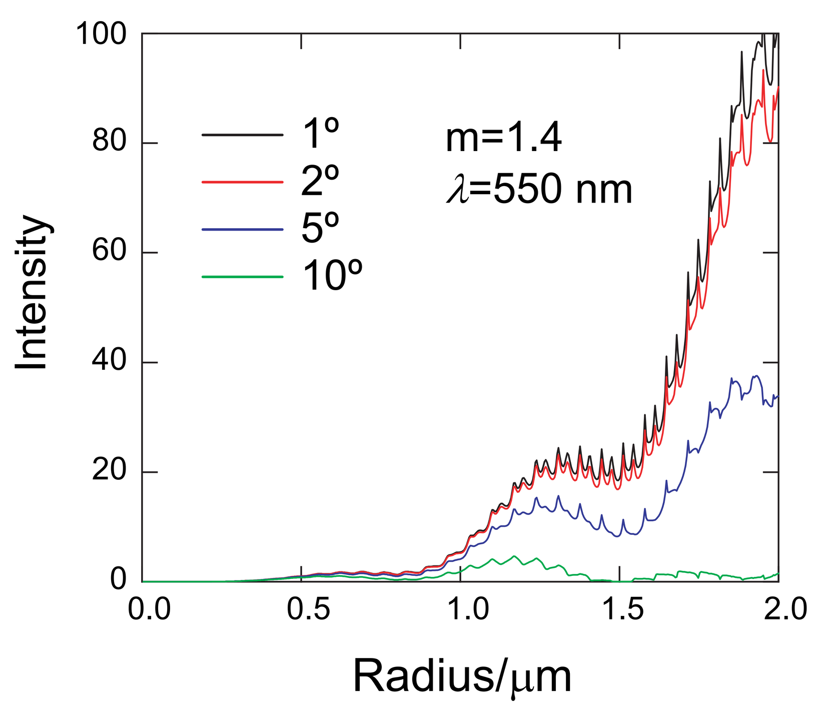
TRANSMITTANCE MEASUREMENTS:

$$\tau_{ep}(\lambda) = \int_0^{TOA} \left(\int_0^\infty \pi r^2 Q_{ext}(m, r/\lambda) \frac{dn}{dr} dr \right) dz$$



SCATTERING MEASUREMENTS:

$$I(\theta,\lambda) \approx \int_0^{TOA} \left(\int_0^\infty \pi r^2 P(\theta, r/\lambda) \frac{dn}{dr} dr \right) \frac{dz}{\cos\psi}$$



- Different measurements yield information about different parts of the size distribution.
- More measurements do not necessarily yield much more information.
- The light-scattering ability of small particles ($r \lesssim 0.25$ μm) is very low.

MOMENTS FROM MEASUREMENTS

As each of the measured quantities is *linear* in the size distribution dn/dr, it is possible to construct linear combinations of measurements that yield moments of the size distribution.

- Define measured quantities by $\phi_j = \int_0^\infty K_j \frac{dn}{dr} dr$
- Radial moments are defined as $\mu_i = \int_0^\infty r^i \frac{dn}{dr} dr$
- Approximating the moments by $\mu_i \approx D_{ij}\phi_j$ is the same as approximating the powers of the radius by $r^i \approx D_{ij} K_j$

The coefficients D_{ij} are determined as follows:

- Construct $C_{ij} = \int_{r_1}^{r_2} K_i K_j dr$
- Construct $P_{ij} = \int_{r_1}^{r_2} r^i K_j dr$
- Then $D_{ij} = P_{il}\left(C_{lj}\right)^{-1}$
- And $\mu_i \approx P_{il}\left(C_{lj}\right)^{-1}\phi_j$

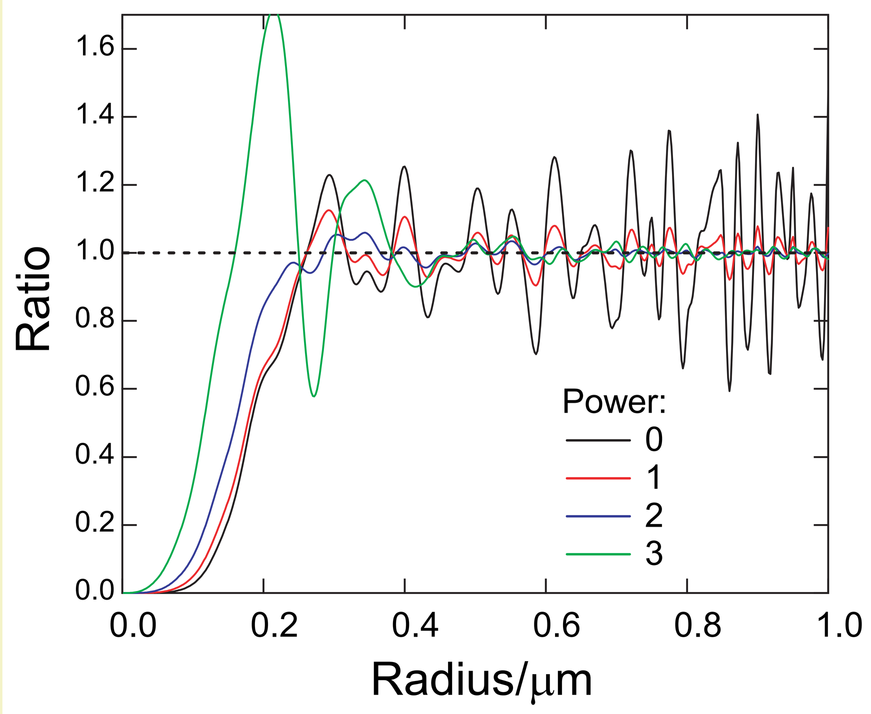
The accuracy with which the moments can be approximated depends on the eigenvalues of C, thus on the independence of the measurements.

PROCEDURE

- Construct approximations to the lowest several radial powers as linear combinations of extinction and scattering kernels at several angles and several wavelengths.
- Approximate moments as integrals over the radius of the size distribution times the reconstructed power.
- Accuracies depend on the size distribution.

RESULTS

- Graph shows the ratio, as a function of radius, of the reconstructed power to the actual power.
- Reconstruction is from measurements of extinction and scattering at several angles at wavelengths 400, 550, & 700 nm.
- For a perfect reconstruction, the ratio should be unity for all radii.



EXAMPLE

- Moments are reconstructed for the aerosol size distribution shown earlier.
- This distribution yields an aerosol optical depth of 0.22 at 550 nm.

	μ_0 cm^{-3}	μ_1 μm cm^{-3}	μ_2 μm^2 cm^{-3}	μ_3 μm^3 cm^{-3}
Reconstructed	479	142	48	19
Actual	662	169	51	18
Ratio	**0.72**	**0.84**	**0.94**	**1.05**

SUMMARY

- The accuracy of the reconstruction of the powers is good for $r \gtrsim 0.25$ μm.
- The lower moments are underestimated somewhat because of the low scattering ability of small particles.