Spectral Irradiance per Interval of Equal Solar Flux: Convenient Spectral Grid for Atmospheric Radiation Measurement and Modeling

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Abstract

Wavelength space can be subdivided into intervals of constant solar flux. This defines a spectral grid, referred to as solar flux numbers in this paper. The spectral irradiance expressed in units of irradiance per solar flux numbers has useful properties: it has no Fraunhofer structure, and is equal to the transmittance because the solar source function is constant in irradiance units per solar flux numbers. This representation of spectral irradiance has utility in atmospheric radiation and modeling by simplifying the quantitative analysis of the energy budget.

Introduction

The spectral irradiance is defined as irradiance per unit wavelength interval at a given wavelength. However, we grew accustomed to a nonlinear subdivision of wavelength space, such as wavenumbers, and express spectral irradiance in $W/m^2/cm^{-1}$. The two physical quantities expressed in $W/m^2/nm$ and $W/m^2/cm^{-1}$ describe the same thing by using slightly different words. Therefore, the definition of spectral irradiance must be broadened to include nonlinear mappings of the wavelength. A natural generalization can be formulated. If a one-to-one differentiable mapping exists between wavelength λ and x, then one can express spectral irradiance in units of irradiance per units of x. This spectral irradiance is equivalent to the spectral irradiance in units of irradiance per wavelength. The two spectral irradiances are equivalent when their integrals over λ and x, respectively, are equal for all possible intervals of integration. We suggest a term of *spectral grid* for any space that is an image of one-to-one differential mapping of wavelength space. The statement that irradiances expressed in different spectral grids are equivalent is corollary to the definition of a spectral grid.

While moving along the electromagnetic spectrum, one encounters keV, eV, Å, nm, µm, cm^{-1}, GHz, m, km, and Hz that measure wavelength in different spectral grids. One can imagine using kelvins to express thermal energy via Boltzman's constant or the color temperature via Wien's law. Each spectral region has a reason to use its own spectral grid. From the aesthetic point of view, it would be pretty if we all used frequency. Frequency is invariant in media; it is proportional to energy; it is the inverse of time. Through the law of causality, one of the most fundamental laws of physics, frequency makes an early appearance in the Kramers-Kronig transform. Like energy, it can be added and subtracted. One can sometimes hear that all spectroscopy can be written without mentioning wavelength even once. While all this is true, nobody really measures frequency directly in the optical region.

It must be emphasized that what we measure and how we measure it has a great impact on units that we choose to use. For instance, in the ultraviolet and visible regions diffraction grating spectroradiometers are the most common. The spectra from these instruments have approximately constant resolution in the wavelength space. On the other hand, Fourier transform spectrometers, which dominate the infrared region, produce spectra with resolution constant in the wavenumber space. A spectrum as an assembly of information is most optimally coded when displayed in units in which the resolution is constant. For this reason, spectra from prism instruments or acousto-optically tuned filters would be displayed most optimally in entirely different units.

In the shortwave region, there are two competing spectral grids: wavelengths and wavenumbers. They usually meet somewhere between 1 and 2 μms where the performance of Fourier transform and grating spectrometers are no longer at their best. Wavenumbers, a much younger relative of frequency, seem to have all the attributes of frequency, and the proportionality to energy is considered most important. However, wavenumbers have one flaw—the pedigree. Their definition as a reciprocal of wavelength creates an opportunity for index of refraction to sneak in through its influence on light velocity. Fourier transform spectrometers measure in the “absolute” wavenumbers only when it is in vacuum. Never-the-less, proportionality to photon energy has its undeniable appeal, particularly to those who are concerned with spectroscopy on the resolution levels of single absorption bands. But in moderate or low-resolution spectroradiometry, the argument of proportionality to photon energy is spurious. However, tying the spectral grid to the energy of radiation being measured led us to investigate a spectral grid defined by the solar source function. This spectral grid we call *solar flux numbers* promises some utility in atmospheric radiation measurements and modeling. In what follows, we define solar flux numbers and investigate their properties.

Intervals of Equal Solar Flux

We find a sequence of wavelengths $0<\lambda_1<\lambda_2<\ldots$ that subdivides the wavelength space into intervals in which the solar flux equals a constant F. Thus, for n=0, 1,…, n_{max}

$$\int_{\lambda_n}^{\lambda_{n+1}} SSF(\lambda)d\lambda = F \qquad (1)$$

For a given solar source function SSF(λ) and the constant F, the resulting sequence of wavelengths is unique. The mapping between indices n and λ_n is one-to-one. This set of indices is a basis of a spectral grid: the solar flux numbers. At this point we are not concerned the grid is discrete and remains undefined on the real numbers continuum. Instead, we proceed with an investigation of the properties of this spectral grid. To facilitate our discussion we assign symbol η_F to the physical quantity of the solar flux number and symbol C_F to the units of this physical quantity (Table 1).

Table 1. Preliminary name and symbols.

Quantity Name	Quantity Symbol	Units Symbol	Units Name
Solar flux number	η_F	C_F	?

The example of usage: If $\eta_F = 10\ C_F$, one describes 10^{th} interval on the basis of F watts-per-square-meter. Thus, $\eta_2 = 365\ C_2$ means the 365^{th} 2 W/m^2 interval of solar flux, which happens to correspond to 760 nm.

Properties of Solar Flux Numbers

(**P1**) Solar source function in irradiance units per solar flux number is constant:

$$SSF(\eta_F) = F\ W/m^2/C_F \tag{2}$$

In Figure 1, the same SSF is presented in three spectral grids.

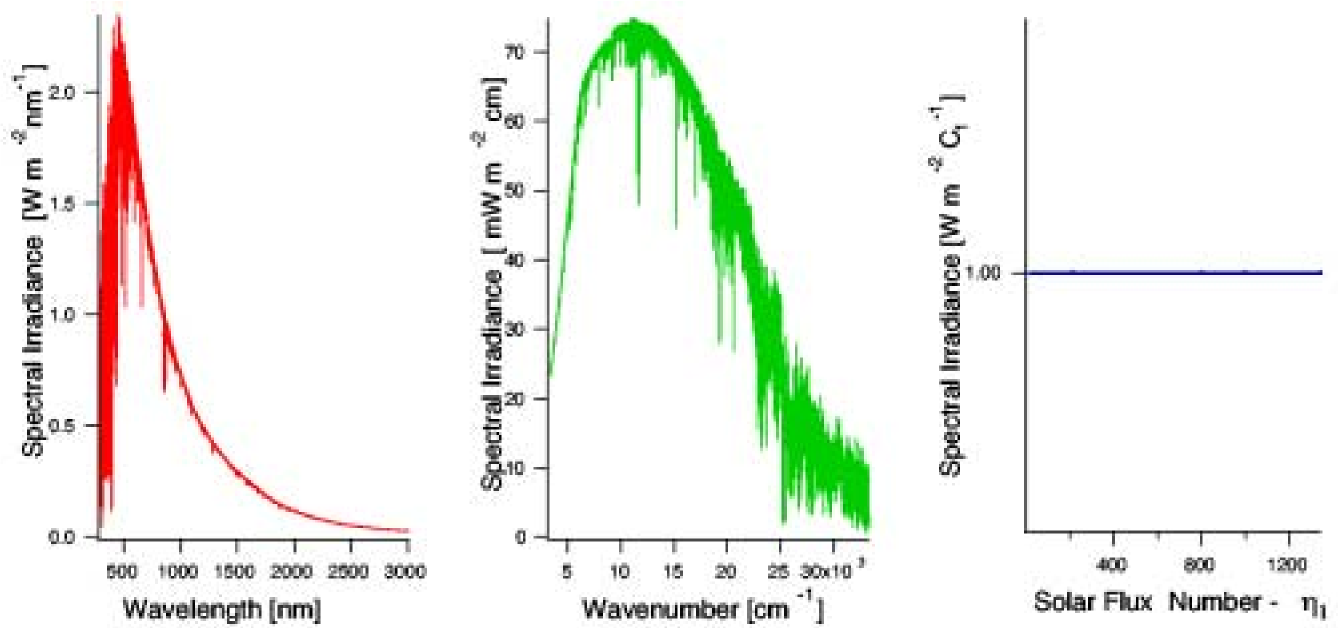

Figure 1. SSF in units of irradiance per wavelength, per wavenumber and per solar flux number (F=1) in the shortwave range (300 to 3000 nm).

(**P2**) The maximum solar flux number is equal to the solar constant divided by F.

$$\max(\eta_F) = SC/F \tag{3}$$

For example, max(η_1)•1370 and max(η_2)•685. In Figure 2, the dependence of η_F on wavelength is depicted for several values of F and Table 2 lists "memorable" wavelengths and respective solar flux numbers.

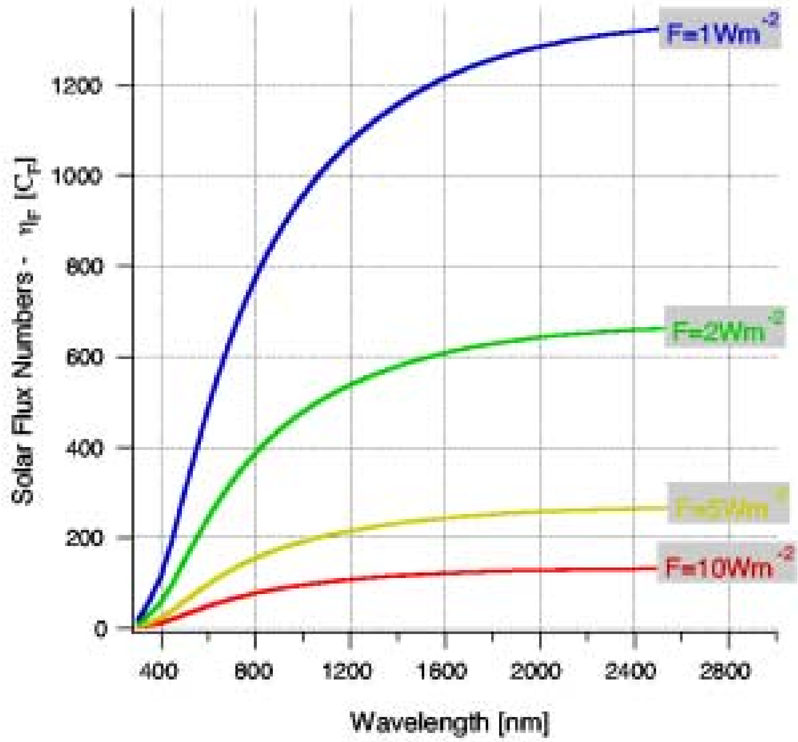

Figure 2. Solar flux numbers dependence on wavelength.

(**P3**) Conversion of monochromatic (or high resolution) spectral irradiance I(λ) in [W/m^2/nm] into spectral irradiance in [W/m^2/C_F] is accomplished with the integral:

$$I(\eta_F) = \int_{\lambda_n}^{\lambda_{n+1}} I(\lambda)d\lambda \tag{4}$$

The resolution of the spectral irradiance $I(\eta_F)$ is constant: full width at half maximum (FWHM) = 1C_F. In Figure 3, the resolution of $I(\eta_F)$ from Eq. (4) in wavelength is plotted against wavelength for several values of F. Obviously, the highest resolution in nanometers occurs at the peak of the solar source function. The curves in Figure 3 can guide us to find the necessary resolution for spectral analysis at a given F-watts-per-square-meter level of precision or magnification. For instance, if one is interested in doing "2 W/m^2 atmospheric science" then FWHM does not need to be better than 1-nm anywhere. And in the infrared, at 2 µm, FWHM = 15 nm is sufficient.

(**P4**) If A and B are two different flux values, then η_A is related to η_B as follows:

$$A\eta_A = B\eta_B \tag{5}$$

Table 2. List of "memorable" wavelengths.

Memorable Wavelengths	Nanometers	F = 1	F = 2
Hg Line	297	16.4	8.2
Ozone Cut-Off	300	18.2	9.1
HeCd Laser Line	325	36.4	18.2
Fraunhofer K Line (Ca)	393	106.5	53.3
Fraunhofer G Line (Fe,Ca)	430	165.4	82.7
HeCd Laser Line	441	183.2	91.6
Max Solar Flux Number Resolution	450	200.5	100.2
Fraunhofer F Line (H)	486	274.8	137.4
Photopic Maximum	555	405.7	202.9
Fraunhofer D1 Line (Na)	589	468.7	234.4
Maximum of Chappuis Band Absorption	603	493.7	246.9
H_2O	720	675.4	337.7
HeNe Line	633	544.6	272.3
O_2	760	727.4	363.7
H_2O	820	797.2	398.6
H_2O	940	909.7	454.9
Hg Line	1014	966.3	483.1
H_2O	1300	1120.2	560.1
H_2O	1900	1272.6	636.3
H_2O and CO_2	2800	1336.8	668.4
End of Shortwave	3000	1342.7	671.4
End of Light	Inf	1370.0	685.0

For example, $365C_2 = 730C_1 \bullet 760$ nm. One can display spectral irradiances with different F against the same abscissa by introducing fractions of solar constant:

$$\mathrm{scf} = (F/SC)\,\eta_F \tag{6}$$

where SC is solar constant. In Figure 4, five spectra for F = 1W/m^2,…, 5W/m^2 are plotted against scf. Each of the spectra has a constant resolution that in scf units is proportional to F. Thus, F is a measure of resolution.

(**P5**) Spectral irradiance in [W/ m^2/C_F] divided by F equals transmittance:

$$I(\eta_F)/F = T(\eta_F) \tag{7}$$

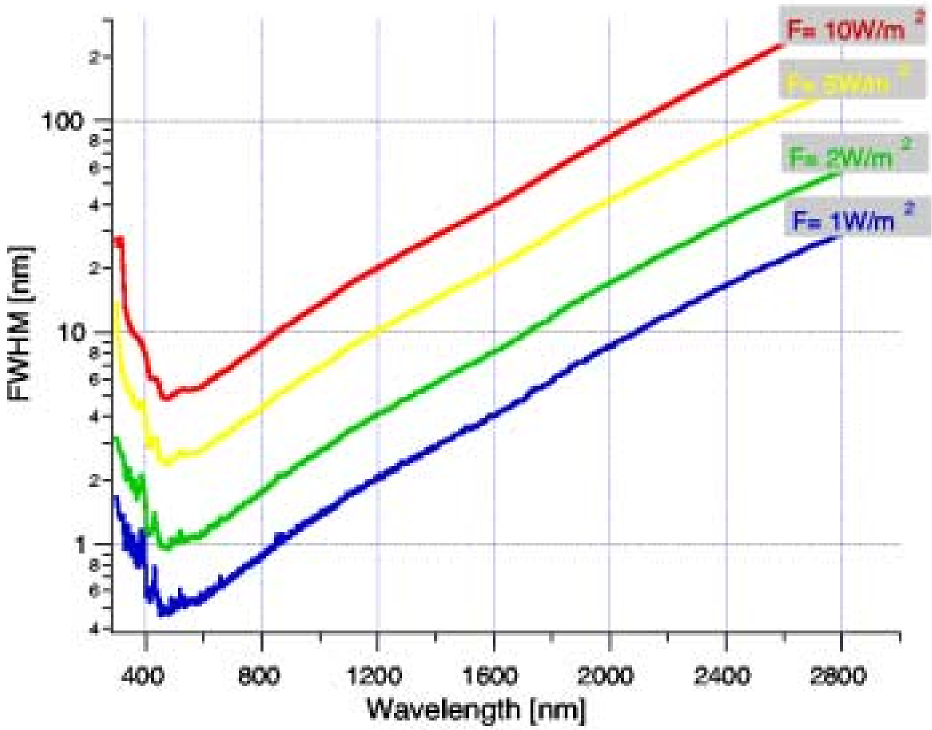

Figure 3. Resolution of spectral irradiance for different values of solar flux F.

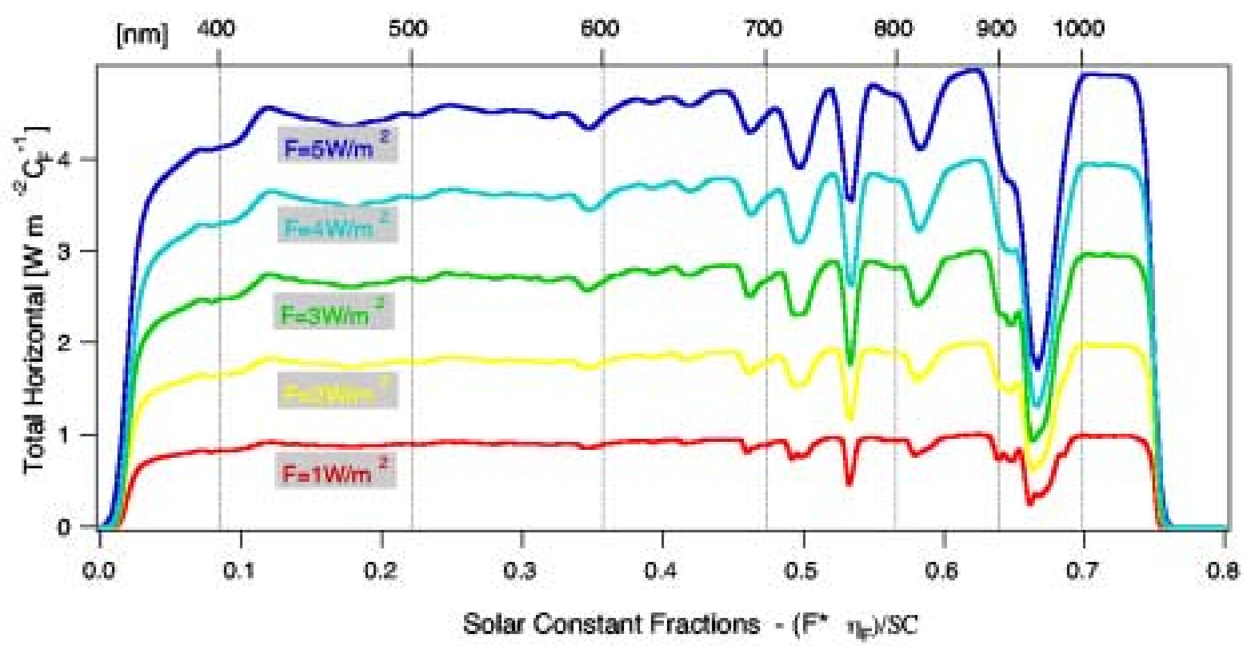

Figure 4. Total horizontal spectral irradiance from MODTRAN is plotted in five different resolutions defined by F.

As a consequence, spectral irradiance in [W/m^2/C$_F$] has no Fraunhofer structure. In Figure 5, we present direct and diffuse spectral irradiances obtained with the Rotating Shadowband Spectroradiometer (RSS) at SGP during the ARM Enhanced Shortwave Experiment (ARESE) II campaign on March 20, 2000, at 17:26 Greenwich Mean Time. They are expressed in irradiance per solar flux numbers. The resolution of the RSS is comparable to F = 2-W/m^2 in solar flux numbers; therefore, units C$_2$ are justified.

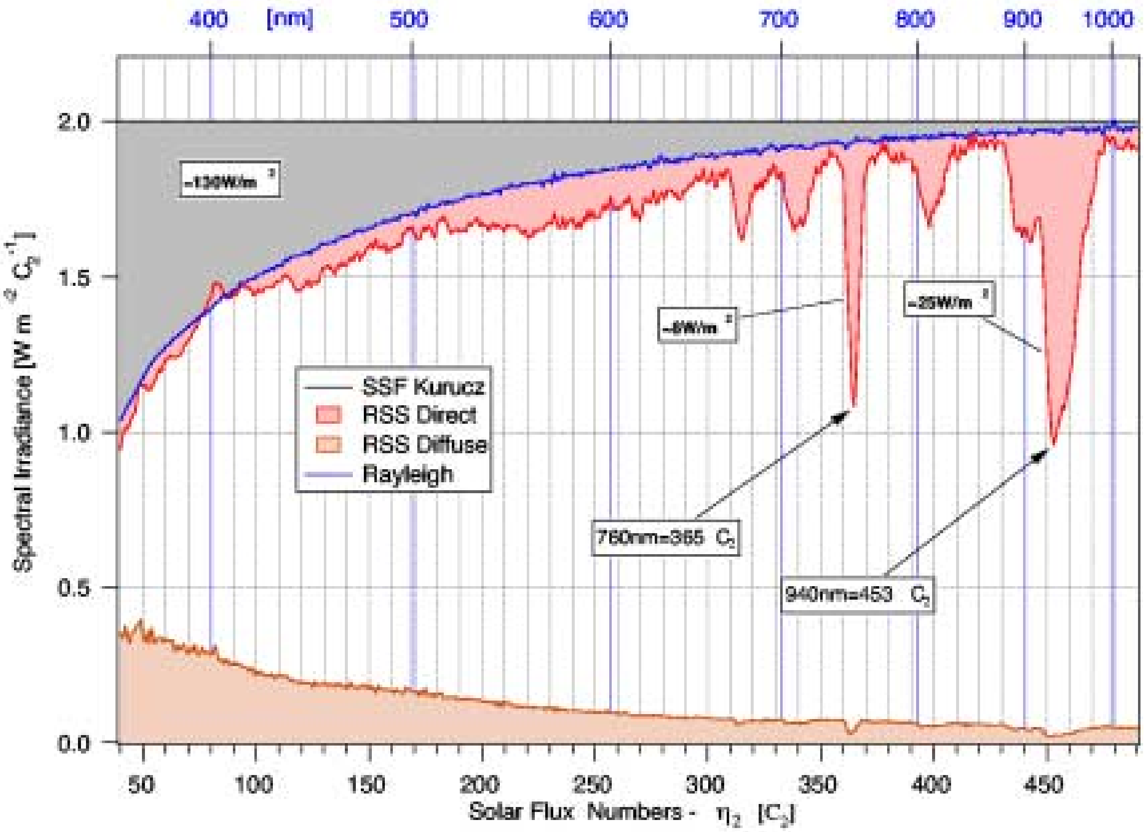

Figure 5. Direct and diffuse spectral irradiance from RSS in W/ m^2/C$_2$.

The fact that the spectral irradiance is equal to the transmittance, allows us to estimate energy integrals directly from the graph. Irradiance absorbed by the 940-nm water band is about 25 W/m^2 and by 760-nm oxygen band is approximately 8 W/m^2 and Rayleigh scattering removes about 130 W/m^2 in the range of the RSS. The first result is obtained as follows: we approximate the 940-nm band with a triangle that has base equal to 50C$_2$ and height equals to F-1 = 1W/m^2/C$_2$; then $0.5 \times (50\ C_2) \times (1\ W/m^2/C_2) = 25\ W/m^2$.

In Figure 6, we show three different solar source functions in W/m^2/C$_2$ where the solar flux numbers are based on Kurucz's SSF. The graphs show that solar flux numbers are quite useful in identifying and assigning energy difference between different solar source functions. Such an immediate quantitative display of energy budget cannot be accomplished if one calculates ratios of spectral irradiances in W/m^2/nm or W/m^2/cm^{-1}.

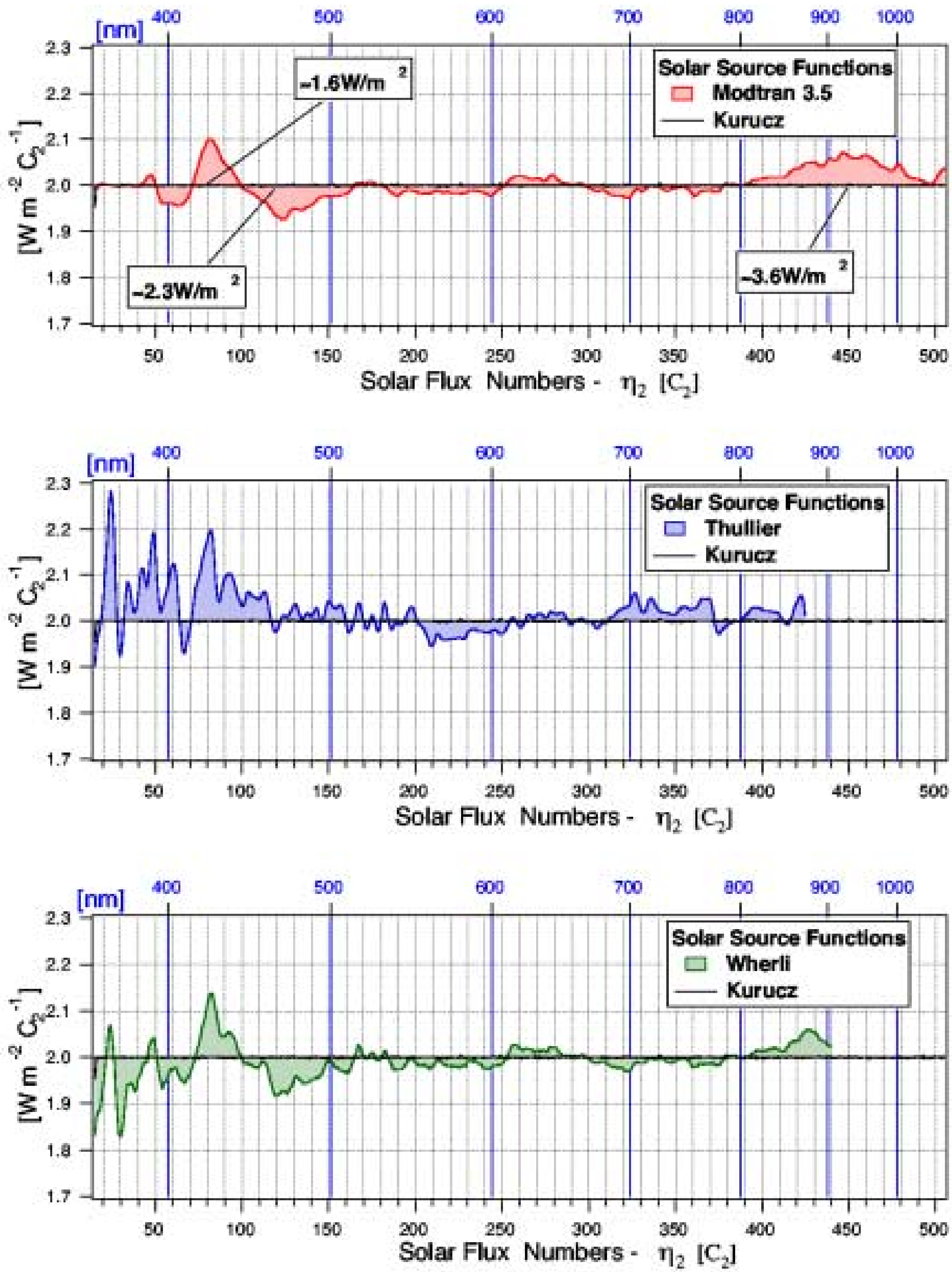

Figure 6. Three solar source functions in W/ m^2/C_2. Solar flux numbers η_2 are defined by Kurucz's solar source function.

A Final Note on Notation

Obviously, the solar flux numbers are not universal. In terms of the universe they are quite provincial and parochial. In fact, they are quite heliocentric. For that reason, I chose η_F since it is the first letter for the sun in Greek. However, all Greek letters are taken in physics and elsewhere, so I would rather use the symbol ह_F if I had such a font. The symbol ह is the last letter in the Sanskrit alphabet (in Kyoto-Harvard convention). Its sound is "ha" and it also happens to mean the sun. For units, I used the letter C thinking of Copernicus who was a rather heliocentric guy.

Appendix: Formal Definition of Solar Flux Numbers

Let us define the mapping H_F: $\eta_F \rightarrow \lambda$ on integers by induction:

For $\eta_F = 1,...,\max(\eta_F)$

$$\text{For } H_F(1) = \lambda \qquad \text{where} \qquad \int_0^{\lambda} SSF(\lambda)d\lambda = F$$

$$\text{For } H_F(\eta_F+1) = \lambda \qquad \text{where} \qquad \int_{H_F(\eta_F)}^{\lambda} SSF(\lambda)d\lambda = F$$

The mapping H_F is extended on the continuum as follows: for any real η_F such that $1 < \eta_F < \max(\eta_F)$ we find integer η_X and a real number X that satisfy $\eta_F = \eta_X(X/F)$. The mapping H_X is defined on integers and $H_X(\eta_X)$ exists. Then we set $H_F(\eta_F) = H_X(\eta_X)$. H_F is differentiable when SSF is continuous.

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