Retrieval of the Cloud Droplet Effective Radius Profile in Stratiform Clouds

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Introduction

The droplet effective radius r_e defined as

$$r_e = \frac{\int r^3 n(r) dr}{\int r^2 n(r) dr}, \tag{1}$$

where n(r) is the droplet size distribution (DSD). r_e is an important parameter in cloud-radiation parameterizations in mesoscale and general circulation models. Although it is recognized that the vertical variations of r_e may significantly affect outcome of such parameterizations, an assumption of constant r_e is commonly used in today's models. This simplification is caused in part by the limited data available on the r_e-height dependence obtained primarily from expensive aircraft sampling of a relatively small number of clouds. It is, therefore, important to be able to determine the vertical profile of r_e using remote sensing that has a potential to sample much larger cloud volumes.

In this study, an algorithm for retrieving r_e from radar reflectivity factor, Z, liquid water path, P, and cloud visible optical depth, τ, is presented. The algorithm performance is evaluated using numerically simulated three-dimensional (3-D) stratiform cloud fields and observation at the Southern Great Plains (SGP) Central Facility (CF) on April 30, 1994. The effect of measurement errors on the retrieval performance and related problems of algorithm testing are also discussed.

Algorithm

By assuming a power law relationship between the cloud liquid water content W and radar reflectivity factor Z in the form

$$Z = aW^b, \tag{2}$$

where a and b are constant in the vertical, the average W profile in stratiform clouds can be accurately retrieved using Z measurements from a vertically pointing millimeter wavelength cloud radar (MMCR), provided that the liquid water path P is also known (Ovtchinnikov and Kogan 2000). This can be done by averaging individual profiles calculated from

$$W(h_i) = \frac{P}{\Delta h \sum_i [Z(h_i)]^{1/b}} [Z(h_i)]^{1/b}, \tag{3}$$

where the summation is performed over the depth of the cloud layer and Δh is the distance between adjacent range gates of the radar, h_i.

Similarly, by assuming a power law relationship between Z and the extinction coefficient ε in the form

$$Z = c\varepsilon^d, \tag{4}$$

the vertical profile of ε can be obtained from Z and the cloud optical depth τ:

$$\varepsilon(h_i) = \frac{\tau}{\Delta h \sum_i [Z(h_i)]^{1/d}} [Z(h_i)]^{1/d} \tag{5}$$

Since W is a function of the third and ε is a function of the second moment of the DSD, r_e can be expressed as

$$r_e = \frac{3Q_e}{4\rho_w} \frac{W}{\varepsilon}, \tag{6}$$

where ρ_w is the density of liquid water and Q_e is the extinction efficiency of the droplets assumed a constant independent of droplet size. Using Eq. (6), the retrievals Eq. (3) and Eq. (5) can be combined to give the profile of r_e:

$$r_e(h_j) = \frac{3Q_e}{4\rho_w} \frac{P}{\tau} \frac{\sum_i [Z(h_i)]^{1/d}}{\sum_i [Z(h_i)]^{1/b}} [Z(h_j)]^{1/b-1/d} \tag{7}$$

Using the result from the Cooperative Institute of Mesoscale Meteorological Studies (CIMMS) large-eddy simulation (LES) model, Ovtchinnikov and Kogan (2000) found that Eq. (3) produced the best retrieval when b = 1.32. Using a similar approach, we find that d = 1.75 (1/d = 0.57) works best for the retrieval of using Eq. (5) (Figure 1).

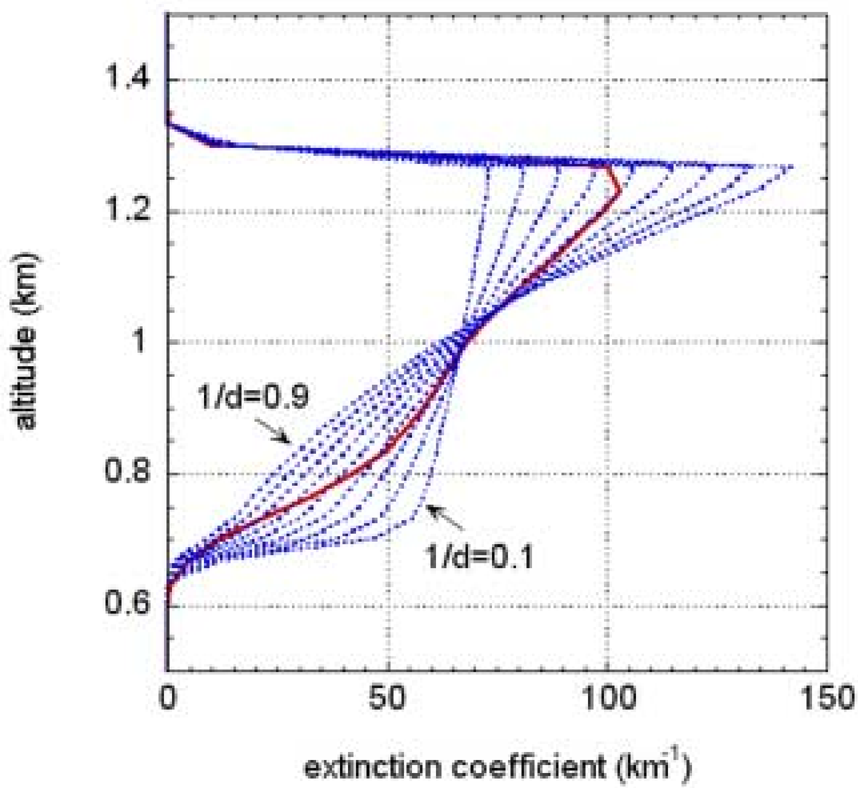

Figure 1. Retrieved average profiles of the extinction coefficient for the nine values of 1/d (see Eq. 5) at 0.1 interval. The benchmark profile calculated directly from simulated DSD is shown in red.

Lognormal Droplet Size Distribution

It is possible to derive an analytical r_e-Z relation by assuming the lognormal droplet size distribution in the form

$$n(r) = \frac{N}{r\sigma_{\ln r}\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln r - \ln r_m}{\sigma_{\ln r}}\right)^2\right], \tag{8}$$

where N is the total concentration, σ_{lnr} is the logarithmic width of the distribution, and r and r_m are the radius and median radius, respectively. The p-th moment of the distribution, M_p, is given by

$$M_p = \int_{r=0}^{\infty} r^p n(r)dr = Nr_m^p \exp\left(\frac{p^2\sigma_{\ln r}^2}{2}\right). \tag{9}$$

Consequently, r_e, ε, W, and Z are readily available:

$$r_e = r_m \exp\left(\frac{5\sigma_{\ln r}^2}{2}\right), \tag{10}$$

$$\varepsilon = \pi Q_e N r_m^2 \exp\left(2\sigma_{\ln r}^2\right), \tag{11}$$

$$W = \frac{4\pi}{3}\rho_w N r_m^3 \exp\left(\frac{9\sigma_{\ln r}^2}{2}\right), \tag{12}$$

$$Z = 2^6 N r_m^6 \exp\left(18\sigma_{\ln r}^2\right). \tag{13}$$

Using Eqs. (10)-(13), we can write

$$Z = \frac{36}{\pi^2 \rho_w^2} \frac{\exp\left(9\sigma_{\ln r}^2\right)}{N} W^2 \tag{14}$$

and

$$Z = \frac{2^6}{\pi^3 Q_e^3} \frac{\exp\left(12\sigma_{\ln r}^2\right)}{N^2} \varepsilon^3. \tag{15}$$

Under approximation of constant $N^{-1}\exp(9\sigma^2_{\ln r})$ and $N^{-2}\exp(12\sigma^2_{\ln r})$, Eqs. (14) and (15) are equivalent to Eqs. (2) and (4), respectively, with b = 2 and d = 3.

Algorithm Performance on a Simulated Data Set

The LES model provides spectral microphysical data for calculations of cloud properties and radar characteristics to which retrieval algorithms are applied. The retrieval is performed for each vertical column and an average of 1600 profiles corresponding to 40×40 horizontal grid cells in the model is calculated. Retrieved average profile of r_e is compared to the one calculated directly from cloud drop spectra. The results of the retrieval are presented in Figure 2. The difference between the two considered algorithms is insignificant; both give an accurate result.

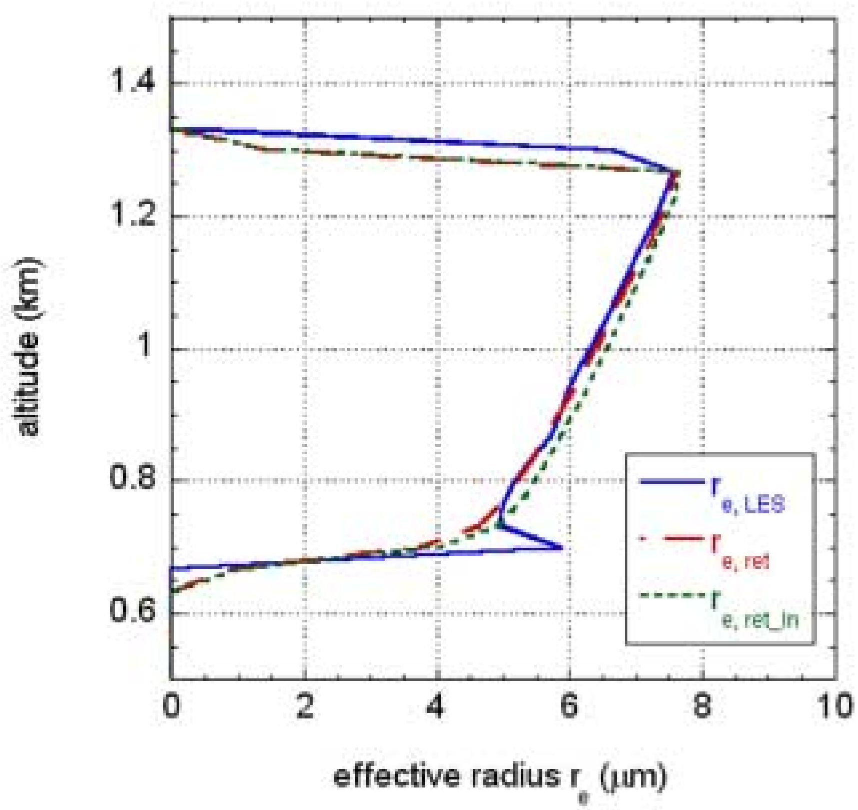

Figure 2. Average effective radius profiles retrieved from a simulated reflectivity field using Eq. (7) with two sets of coefficients, b and d, based on present work (blue) and lognormal approximation (red). The benchmark profile calculated directly from simulated DSD is shown in blue.

Empirical Algorithm

By calculating Z and r_e from cloud droplet spectra measured by the forward scattering spectrometer probe (FSSP), Fox and Illingworth (1997) found that the relation of the form

$$Z \sim r_e^f \tag{16}$$

is best fitted to points when f = 4.09 although the scatter on a Z - r_e plot was quite large. For r_e averaged over 2.5-dBZ bins, they found that f = 5.65 ensures higher correlation. We can relate Eq. (16) to Eq. (7) by setting f = bd/(b-d). Thus, for our algorithm f = 5.37 (b = 1.32 and d = 1.75), while under the lognormal approximation f = 6 (b = 2 and d = 3). Fox and Illingworth's estimate lies between these values and, therefore, is equally appropriate.

Comparison with In-Situ Measurements

For the April 30, 1994, case, the retrieval was performed using reflectivity measurements collected by the University of Massachusetts Cloud Profiling Radar System. P was derived using aircraft data. We used τ derived from the Geostationary Operational Environmental Satellite (GOES)-7 measurements over the SGP CF (36.61N, 97.49W). The average values of $P_{avr} = 150$ g m^{-2} and $\tau_{avr} = 28$ are used in the retrievals presented in Figure 3. Cloud droplet spectra collected by the University of North Dakota (UND) Citation aircraft were used to calculate r_e directly. Shown are the r_e values for the 2040 Universal Time Coordinates (UTC) to 2130 UTC interval and average retrieved profiles for the same period.

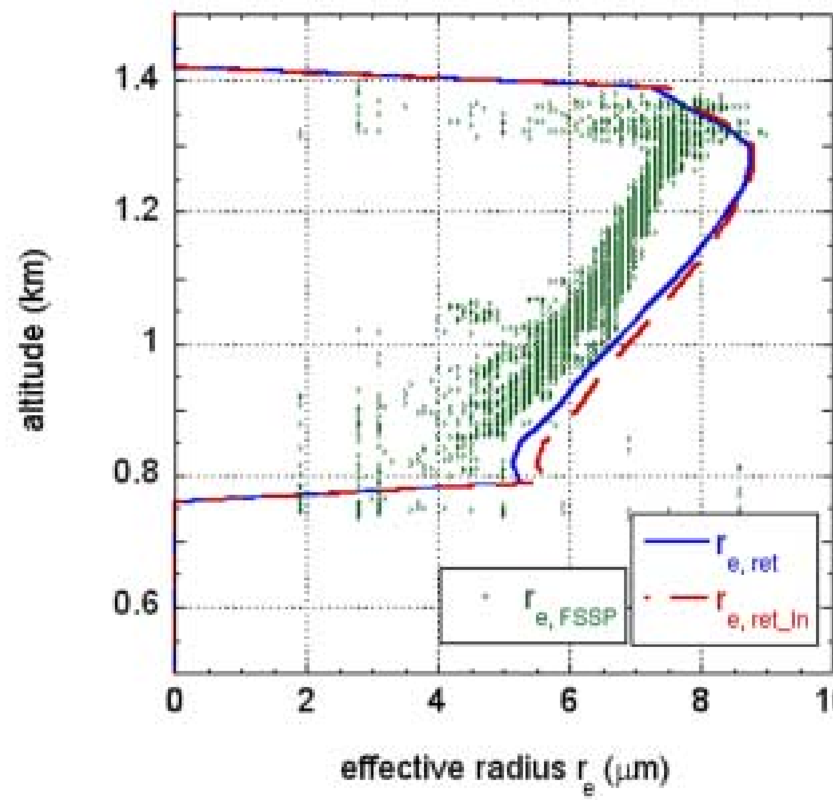

Figure 3. Retrieved average r_e profiles using Eq. (7) with two sets of coefficients, b and d, based on present work (blue) and lognormal approximation (red). Values calculated directly from FSSP-measured spectra are shown in green.

The retrieved r_e overestimates the measured values by 0.5 to 1.5 μm, which is likely due to underestimated τ (see Figure 4). There is a pronounced increase of r_e by about 4 μm from cloud base to cloud top. Again, we note that the results are not sensitive to the exponent in Eq. (7) as the blue and red curves in Figure 3 are very close to each other.

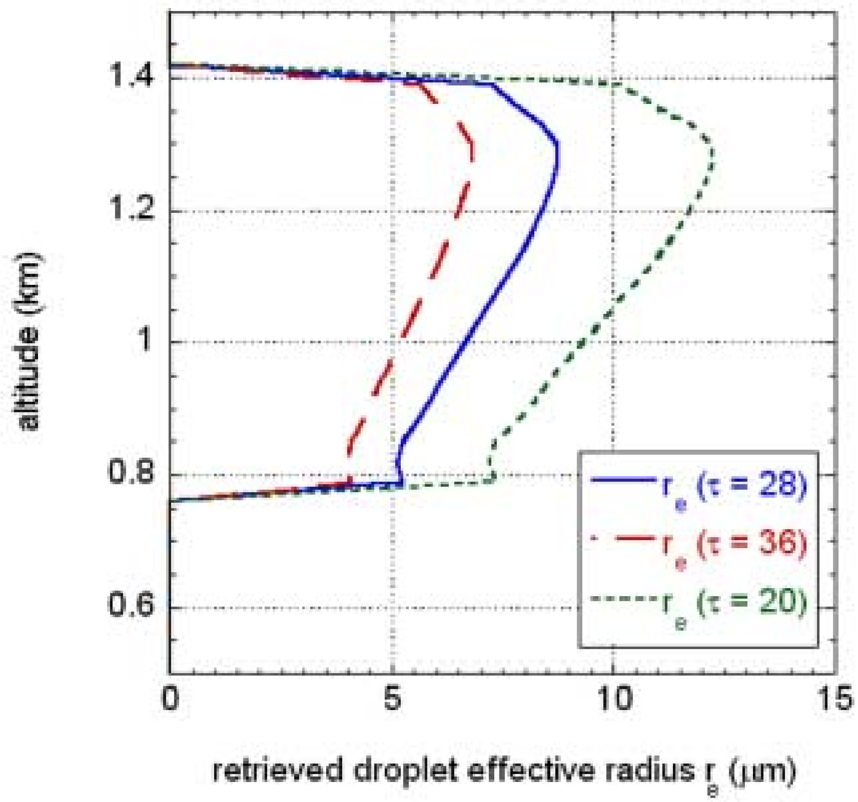

Figure 4. Retrieved average profiles of the effective radius for three values of the cloud optical depth.

Discussion

According to Eq. (7), the retrieved r_e is inversely proportional to the cloud optical depth and directly proportional to the liquid water path. Consequently, the method requires accurate estimates of these parameters. The sensitivity of r_e to the uncertainties in τ is illustrated in Figure 4 that shows the retrieved profiles using the average optical depth $\tau_{avr} = 28$ as well as this value plus and minus one standard deviation $\Delta\tau = 7.65$ derived from GOES-7 measurements. It is clear that the uncertainties of this magnitude have a profound effect on the retrieved profiles.

Uncertainties in P are equally important. The difference between P retrieved from microwave radiometer (MWR) and from aircraft data was quite significant (up to 100% at times) on April 30, 1994, although the source of the error was not identified (Sassen et al. 1999). This precluded the use of the MWR data in the case study.

Validation of microphysical retrievals using aircraft measurements remains a problem, primarily because of the enormous disparity in sampled cloud volumes and uncertainties in space-time coordination of measurements. For example, spirals performed by the UND Citation aircraft around the CF on April 30, 1994, were about 10 km in radius. Coincidentally, one of the most prominent features of the cloud structure (fluctuations of the cloud depth and liquid water path) were observed on the same scale. Furthermore, during even the fastest ramps attempted during the mission (ascent/descent rate of ~10 m m^{-2}), the aircraft covers within a cloud a horizontal distance of several kilometers. These and other sampling- and averaging-related issues must be taken into account in all retrieval validation studies.

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