A Generalization of Stochastic Radiative Transfer Model: Multiple Broken Layers

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Introduction

The problem of stochastic radiative transfer in clouds has been aimed at establishing the relationship between the statistical parameters of clouds and radiation. Although the necessity of such treatment was acknowledged long ago and several approaches were suggested (see, e.g., Cahalan et al. 1994; Gabriel and Evans 1996; Lovejoy et al. 1990; Stephens et al. 1991), the solution of the complex problem as a whole is still far from completed.

A promising approach to describe the stochastic radiative transfer in broken clouds is to use statistically homogeneous models (see, e.g., Titov 1990; Malvagi et al. 1993). These models have been used to treat correctly the horizontal inhomogeneity of broken cloud fields and can be applied easily to multilayer systems, consisting of statistically independent layers of broken clouds (the hypothesis of random cloud overlap). However, this hypothesis could be inappropriate (see, e.g., Barker et al. 1999) since the spatial distributions of clouds at individual atmospheric levels may depend on how the clouds are distributed over other levels.

An approach (Titov and Kassianov 1999) that can be used to describe the radiation interaction with multiple interrelated fields of broken clouds is presented. This approach is based on the stochastic transfer equation and a new statistically inhomogeneous model.

Statistically Inhomogeneous Model

The broken clouds can be represented as a Markovian mixture of cloudy and noncloudy segments. In this case, random indicator field $\kappa(\mathbf{r})$ is described completely by unconditional $\langle\kappa(\mathbf{r})\rangle$ and conditional $V(\mathbf{r},\mathbf{r}')=P\{\kappa(\mathbf{r})=1/\,\kappa(\mathbf{r}')=1\}$ probabilities of the cloud presence. The angular brackets here are for ensemble averages over the cloud field realizations. For the statistically homogeneous models, the unconditional probability $\langle\kappa(\mathbf{r})\rangle$ does not depend on the vertical coordinate, and the conditional one $V(\mathbf{r},\,\mathbf{r}')$ depends only on the distance between points $\mathbf{r}$ and $\mathbf{r}'=\mathbf{r}+\boldsymbol{\omega}(\xi-z)/\,c$ (for a fixed direction $\boldsymbol{\omega}$).

In real single- and multi-layer broken clouds, the statistical parameters, such as $\langle \kappa(\mathbf{r}) \rangle$, can vary strongly with altitude, and values of these parameters at different altitudes can be interrelated. In this connection, a new statistically *inhomogeneous* model for such broken clouds has been constructed (Titov and Kassianov 1999). The term "statistical inhomogeneity" is understood in the meaning that the unconditional probability $\langle \kappa(\mathbf{r}) \rangle$ depends on the vertical coordinate and the conditional probability $V(\mathbf{r}, \mathbf{r}')$ depends on the positions of points $\mathbf{r}$ and $\mathbf{r}'$.

To obtain the conditional probability $V(\mathbf{r}, \mathbf{r}')$ of the cloud presence, we will use the piecewise constant approximation of probabilities of transition from cloud to clear sky and backward. It is assumed that the probability of transition, on a short distance $\Delta l = \Delta z / c$, from clear sky to cloud $0 \to 1$ is $\mu_i \Delta l$ and from cloud to clear sky $1 \to 0$ is $\eta_i \Delta l$, $i = 1, ..., M$, where M is the number of layers. Note that $1/\mu$ and $1/\eta$ can be interpreted as mean chord lengths in the cloud-free and cloudy segments, respectively, and they can depend on the direction ω (Malvagi et al. 1993). The conditional probability of cloud presence $V(\mathbf{r}, \mathbf{r}')$ satisfies the Chapman–Kolmogorov equation

$$\frac{\partial V(l)}{\partial l} = -(\mu + \eta)\, V(l) + \mu, \quad V(0) = 1 \tag{1}$$

where $l = |\mathbf{r} - \mathbf{r}'|$, and μ and η are piecewise constant functions of z. The general solution of Eq. (1) has the form

$$V(l) = \exp\left(-\int_0^l (\mu(u) + \eta(u))\, du \right) + \int_0^l \mu(v) \exp\left(-\int_\nu^l (\mu(u) + \eta(u))\, du \right) dv \tag{2}$$

If the points $\mathbf{r}$ and $\mathbf{r}'$ belong to the same ith layer, $i = 1, ..., M$, then using Eq. (2), we obtain

$$V_i(z, \xi) = (1 - p_i) \exp\left(-A_i \frac{z - \xi}{c} \right) + p_i, \tag{3}$$

$$p_i = \frac{\mu_i}{\eta_i + \mu_i}, \qquad A_i = \eta_i + \mu_i, \tag{4}$$

where $i = 1, ..., M$, M is number of layers. Parameter p_i, $i = 1, ..., M$ can be interpreted as the cloud fraction in the ith layer; and the parameter A_i, $i = 1, ..., M$ is inversely proportional to its correlation length (Malvagi et al. 1993). Note, that, for the statistically homogeneous model (Titov 1990), the conditional probability of the cloud occurrence is given by formula (3).

Let the points $\mathbf{r}$ and $\mathbf{r}'$ belong to the different layers, namely the *j*th and the *i*th (*j*>*i*) layers, $i = 1, \ldots, M-1$ and $j = 2, \ldots, M$. Then, using Eq. (2), we obtain the recursion formula for the conditional probability $V_{j,i}$, which determines the statistical relationship between the different layers, has the form

$$V_{j,\,i}\,(z,\xi) = \exp\left(-A_j\,\frac{z - z_{j-1}}{c}\right) \times \left\{V_{j-1,\,i}\,(z_{j-1},\xi) - p_j\right\} + p_j\ , \tag{5a}$$

if $j - 1 = i$, then

$$\begin{aligned} &V_{j-1,\,i}\,(z_{j-1},\xi) = V_i\,(z_i,\xi);\\ &i = 1, \ldots, M-1; \quad j = 2, \ldots, M. \end{aligned} \tag{5b}$$

Let us note some appealing features of the constructed model. The multilevel cloud systems may contain statistically dependent broken-cloud layers; so, different combinations of maximum and random cloud overlaps are normally used in general circulation models. The suggested statistically inhomogeneous model allows one to describe these different situations. For instance, for a random cloud overlap, it is necessary to set $V_{j,\,j-1} = p_j$, $j = 2, \ldots, M$. For a maximum cloud overlap, it should be assumed that $V_{j,\,j-1} = p_j \,/\, p_{j-1}$ for $p_j < p_{j-1}$, $j = 2, \ldots, M$, or that $V_{j,\,j-1} = 1$ for $p_j \geq p_{j-1}$, $j = 2, \ldots, M$. The sketched flexibility of the suggested statistically inhomogeneous model is its important advantage. Further, the relatively few input parameters of the model can be obtained from cloud observations of real cloud fields. This allows one to make a correct comparison of theory against experiment. Suppose that the unconditional probabilities p_i, $i = 1, \ldots, M$, and conditional V_i, $V_{j,i}$, $i = 1, \ldots, M-1$, $j = 2, \ldots, M$ probabilities are known from field observations. Then, from Eqs. (3) and (5a,b) we can determine the unknown parameter A_i, $i = 1, \ldots, M$. Finally, the constructed model is the generalization of the statistically homogeneous models, and therefore, makes it possible to derive and solve equations for the mean intensity of solar radiation in statistically inhomogeneous broken clouds by the methods developed for the statistically homogeneous models. The latter is illustrated in the next section.

Equations for the Mean Intensity

The mean intensity $\langle I(\mathbf{r},\boldsymbol{\omega})\rangle$ of solar radiation can be written as a sum of the unscattered $\langle j(\mathbf{r},\boldsymbol{\omega})\rangle$ and the diffuse $\langle i(\mathbf{r},\boldsymbol{\omega})\rangle$ components. The closed equations for the mean intensity of unscattered radiation $\langle j(\mathbf{r},\boldsymbol{\omega})\rangle$ and the function $\langle \kappa(\mathbf{r})\, j(\mathbf{r},\boldsymbol{\omega})\rangle$ have been derived and solved (Titov and Kassianov 1999) by the methods developed for the statistically homogeneous models. Here we will apply these methods for obtaining and solving analogous equations for the mean intensity of diffuse radiation.

We will assume that the cloud optical properties, namely, the extinction coefficient $\sigma(\mathbf{r})$, the single scattering albedo $\omega_0(\mathbf{r})$, the scattering phase function $G(\mathbf{r},\boldsymbol{\omega},\boldsymbol{\omega}')$ are constant in each layer (the piecewise constant approximation). For simplicity, we start our consideration with a cloud field composed of two layers of broken clouds. The extension to three and more layers ($M > 2$) will be evident. The reflection from the underlying surface and the scattering and absorption of solar radiation by aerosol and atmospheric gases will not be considered. We will assume that parallel flux of solar radiation incident at the upper boundary of second cloud layer in direction $\boldsymbol{\omega}_{\oplus}$.

Suppose we need to determine the mean intensity of reflected ($c > 0$) solar radiation in the second ($z_1 \le z \le z_2$) layer. The random intensity $i(\mathbf{r},\boldsymbol{\omega})$ satisfies the stochastic radiative transfer equation

$$i(\mathbf{r},\boldsymbol{\omega}) + \frac{\sigma_2}{c}\int_{z_1}^{z} \kappa(\mathbf{r}')\, i(\mathbf{r}',\boldsymbol{\omega})\, d\xi = \frac{\sigma_2}{c}\int_{z_1}^{z} \kappa(\mathbf{r}')\, B(\mathbf{r}',\boldsymbol{\omega})\, d\xi + i(\mathbf{r}_1,\boldsymbol{\omega}) \tag{6}$$

$$i(\mathbf{r}_1,\boldsymbol{\omega}) + \frac{\sigma_1}{c}\int_{z_0}^{z_1} \kappa(\mathbf{r}')\, i(\mathbf{r}',\boldsymbol{\omega})\, d\xi = \frac{\sigma_1}{c}\int_{z_0}^{z_1} \kappa(\mathbf{r}')\, B(\mathbf{r}',\boldsymbol{\omega})\, d\xi \tag{7}$$

$$B(\mathbf{r},\boldsymbol{\omega}) = \omega_0(\mathbf{r}) \times \left\{ \int_{4\pi} G(\mathbf{r},\boldsymbol{\omega},\boldsymbol{\omega}')\, i(\mathbf{r},\boldsymbol{\omega}')\, d\boldsymbol{\omega}' + \int_{4\pi} G(\mathbf{r},\boldsymbol{\omega},\boldsymbol{\omega}')\, j(\mathbf{r})\, \delta(\boldsymbol{\omega} - \boldsymbol{\omega}_{\oplus})\, d\boldsymbol{\omega}' \right\} \tag{8}$$

We average Eq. (6) over the ensemble of realization of the field $\kappa(\mathbf{r})$

$$\langle i(\mathbf{r},\boldsymbol{\omega})\rangle + \frac{\sigma_2}{c}\int_{z_1}^{z} \langle \kappa(\mathbf{r}')\, i(\mathbf{r}',\boldsymbol{\omega})\rangle\, d\xi = \frac{\sigma_2}{c}\int_{z_1}^{z} \langle \kappa(\mathbf{r}')\, B(\mathbf{r}',\boldsymbol{\omega})\rangle\, d\xi + \langle i(\mathbf{r}_1,\boldsymbol{\omega})\rangle \tag{9}$$

Function $\langle i(\mathbf{r}_1,\boldsymbol{\omega})\rangle$ is the mean intensity of upward (c>0) diffuse radiation incident upon the lower boundary of the second layer and can be calculated by well-developed algorithms (Titov 1990). Equation (9) is open, since it contains unknown function $\langle \kappa(\mathbf{r}) i(\mathbf{r},\boldsymbol{\omega})\rangle$. To get it, we multiply Eq. (6) by $\kappa(\mathbf{r})$ and average again

$$\langle \kappa(\mathbf{r}) i(\mathbf{r},\boldsymbol{\omega})\rangle + \frac{\sigma_2}{c}\int_{z_1}^{z} \langle \kappa(\mathbf{r})\kappa(\mathbf{r}') i(\mathbf{r}',\boldsymbol{\omega})\rangle\, d\xi = \frac{\sigma_2}{c}\int_{z_1}^{z} \langle \kappa(\mathbf{r})\kappa(\mathbf{r}') B(\mathbf{r}',\boldsymbol{\omega})\rangle\, d\xi + \langle \kappa(\mathbf{r}) i(\mathbf{r}_1,\boldsymbol{\omega})\rangle \tag{10}$$

Applying the constructed model (previous section) and a formula for statistical closure (correlation splitting), we can rewrite Eq. (10) as

$$\langle \kappa(\mathbf{r}) i(\mathbf{r},\boldsymbol{\omega})\rangle + \frac{\sigma_2}{c}\int_{z_1}^{z} V_2(\mathbf{r},\mathbf{r}')\langle \kappa(\mathbf{r}') i(\mathbf{r}',\boldsymbol{\omega})\rangle d\xi = \frac{\sigma_2}{c}\int_{z_1}^{z} V_2(\mathbf{r},\mathbf{r}')\langle \kappa(\mathbf{r}') B(\mathbf{r}',\boldsymbol{\omega})\rangle d\xi + \langle \kappa(\mathbf{r}) i(\mathbf{r}_1,\boldsymbol{\omega})\rangle \quad \text{(10a)}$$

After manipulations similar to those performed when deriving Eq. (10a), we can get the equation for the term $\langle \kappa(\mathbf{r}) i(\mathbf{r}_1,\boldsymbol{\omega})\rangle$

$$\langle \kappa(\mathbf{r}) i(\mathbf{r}_1,\boldsymbol{\omega})\rangle + \frac{\sigma_1}{c}\int_{z_0}^{z_1} V_{2,1}(\mathbf{r},\mathbf{r}')\langle \kappa(\mathbf{r}') i(\mathbf{r}',\boldsymbol{\omega})\rangle d\xi = \frac{\sigma_1}{c}\int_{z_0}^{z_1} V_{2,1}(\mathbf{r},\mathbf{r}')\langle \kappa(\mathbf{r}') B(\mathbf{r}',\boldsymbol{\omega})\rangle d\xi \quad (11)$$

By solving the closed system of Eqs. (9), (10a), and (11), we finally obtain $\langle i(\mathbf{r},\boldsymbol{\omega})\rangle$. The solution can be presented by the form

$$\langle i(\mathbf{r},\mathbf{r}_0)\rangle = \langle i(\mathbf{r},\mathbf{r}_1)\rangle + \langle i(\mathbf{r}_1,\mathbf{r}_0)\rangle \langle j(\mathbf{r},\mathbf{r}_1)\rangle + \left(p_1 u(\mathbf{r}_1,\mathbf{r}_0) - p_2 \langle i(\mathbf{r}_1,\mathbf{r}_0)\rangle\right) \varepsilon(\mathbf{r},\mathbf{r}_1) \quad (12)$$

The first term $\langle i(\mathbf{r},\mathbf{r}_1)\rangle$ in this formula can be interpreted as the mean intensity of diffuse radiation formed in the second layer. The second and the third terms describe the mean intensity of diffuse radiation formed in the first layer and then passed through the second layer without scattering. The functions, entering into Eq. (12), are known and obtained on the basis of the statistically homogeneous model (Titov 1990; Titov and Kassianov 1999). Consequently, we have illustrated that the problem of the solar radiative transfer in the statistically inhomogeneous broken clouds can be successfully solved by the well-known methods developed for the statistically homogeneous models.

Summary

We have introduced an approach for the stochastic description of the solar radiation transfer through broken clouds with the arbitrary inhomogeneity, both the horizontal and vertical. The stochastic radiative transfer and a new statistically inhomogeneous model of broken clouds form the basis of this approach.

The constructed statistically inhomogeneous model has three appealing features. First, the relatively few input parameters of the statistically inhomogeneous model can be derived from observations. This allows one to compare correctly theory predictions with field data. Further, the model flexibility makes it possible to describe the different combinations of random and maximum cloud overlaps, which are used in general circulation models. Finally, the constructed statistically inhomogeneous model is a generalization of the statistically homogeneous models. Consequently, the problem of solar radiative transfer in statistically inhomogeneous broken clouds can be solved by the methods developed for the statistically homogeneous model. Using the mean intensity of diffuse solar radiation as an example, we have demonstrated this possibility.

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