Using a Constrained Linear Inversion Algorithm to Retrieve Cloud Properties from the ARM 35-GHz Radar

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Introduction

There are several problems in atmospheric science, particularly in remote sensing, that involve recovering a signal which has been multiplied by a set of overlapping weighting functions. In such cases, the measured signal at a particular point, $\hat{S}_i$, is given by

$$\hat{S}_i = \int_0^\infty S(j)\,K_i(j)\,dj \tag{1}$$

where S(j) is the true signal and $K_i(j)$ is the i^{th} weighting function evaluated at position j, hereto referred to the kernel function. One example is the broadening of radar Doppler spectra due to turbulent air motions within the radar volume.

Doppler spectra, collected from a vertically pointing radar, are a measure of spectral reflectivity per interval of Doppler velocity. Ideally, the velocity interval can be converted to an equivalent drop size interval via an appropriate size versus velocity relationship. Since radar backscatter from a population of drops depends on the size and number density of the drops, and the size interval is known, the number density can be easily calculated.

However, there is substantial difficulty in converting from the velocity intervals to size intervals. This problem occurs because the drops are embedded within a turbulent medium that alters the measured spectrum in two significant ways: 1) the mean vertical velocity (referred to as the volume mean vertical velocity) shifts the spectrum a uniform amount, and 2) the distribution of turbulent velocities (referred to as the sub-volume scale turbulence) broadens the quiet-air spectrum (i.e., the “true” velocity spectrum with no background air motions). These two scales of vertical velocity must be removed before any size informs can be obtained.

The majority of techniques presented in the literature for removing the turbulent contamination rely on parameterization of the drop distribution and modeling the distribution of vertical velocities in the radar volume (Gossard 1994; Babb et al. 1999). Despite the progress made towards the problem, all of these retrieval techniques assume a three-parameter, liquid-phase, analytical function for the drop distribution (modified gamma or lognormal). This assumption highly limits the application of the techniques to a small subset of observed clouds.

In order to fully solve this problem, a technique must be devised that is suitable for multi-mode, non-analytical hydrometeor spectra. Therefore, no assumptions about the shape of the drop distribution should be incorporated in the retrieval technique. Furthermore, it would be desirable to have a technique that made no assumptions about the phase (liquid, ice, or mixed phase) of the particles in the radar volume. Gossard and Strauch (1989) recognize this and attempt to directly remove the turbulent broadening via both iterative and fast Fourier transform (FFT) deconvolution techniques. However, as is the case with any deconvolution algorithm, noise (uncertainty) in the measured signal and a finite limit in computational precision, severely distort the retrieved signal.

Technique

One approach for improving a deconvolution technique is to incorporate knowledge about the desired solution into the retrieval. This knowledge constrains the solution to a set of reasonable values and does not allow noise in the algorithm to grow unchecked. We found that the best way to incorporate constraints into a deconvolution algorithm was to follow a linear algebra approach to the problem.

Returning to Eq. (1), we see that the integral can be discretized into the following form.

$$\hat{S}_i = \sum_{j=1}^{M} S_j K_{i,j} \Delta j \tag{2}$$

If the unknown quiet-air Doppler spectrum is given by the M intervals of S, and there are N measurements of the contaminated spectrum, $\hat{S}$, then it is clear that there exists a set of N equations and M unknowns. In matrix form this can be written as

$$\hat{S} = K \bullet S \tag{3}$$

where $\hat{S}$ is an N-column vector, S is an M-column vector, and K is the convolution kernel, an N×M matrix. For our purposes we will consider the case where the measured and retrieved velocity resolutions are the same (i.e., M=N) so that the solution of Eq. (3) is simply the inverse of K post-multiplied by $\hat{S}$. We note that there is no reason besides simplicity to make this assumption. For cases where N≠M, it is standard practice to square a system of equations by multiplying both sides of the equation by the transpose of the kernel, K^T.

The solution to Eq. (3) appears simple; however, the convolution kernel is difficult to invert because it is not full rank. This means that the set of equations have less degrees of independence than the number of unknowns. This fact arises because the convolution kernel has the form

$$K_{i,j} = \frac{1}{\sqrt{\pi}\, w_\sigma} \exp\left[-\left(\frac{(i-j)\Delta v}{w_\sigma} \right)^2 \right] \Delta v \tag{4}$$

where w_σ acts to determine the standard deviation (i.e., width) and thus describes the turbulent intensity. Since this function is smooth and Δv is small, the coefficients of a given unknown (e.g., S_j) for two successive equations (e.g., $K_{i,j}$ and $K_{i+1,j}$) are nearly equal to machine precision. The result of K being less than full rank is that once again noise in the measured values and loss of precision during inversion will render the result useless. However, since the deconvolution has been specified in matrix form, we can capitalize on the body of knowledge developed by mathematicians for solving similar problems. Specifically, we will focus on obtaining solutions by adding constraints to the problem.

There are three categories of constraints: equality constraints (e.g., LaGrange Multipliers), one-sided inequality constraints (e.g., non-negative least squares [NNLS] algorithms), and two-sided inequality constraints (e.g., quadratic programming). Since two-sided inequality constraints encompass the other two types, it would be ideal to use them. However, quadratic programming is computationally intensive and is unfeasible for our application. Instead, we will use a sub-optimal variant presented by Pierce and Rust (1985). This method can be illustrated as follows.

Consider an arbitrary confidence ellipsoid (S) for the solution of Eq. (3). Since K is ill conditioned, this S-ellipsoid is greatly elongated in the directions of the eigenvectors corresponding too small eigen values of K^TK (Figure 1a shows a two-dimensional [2-D] example). The elongated ellipsoid and hence large solution space produces results that are far from the true solution. We seek to improve on the bounds of the S-ellipsoid by incorporating knowledge about where the true solution resides.

The bounding of the S-ellipsoid is accomplished by adding an N-dimensional bounding box (Q) where the true solution (Figure 1b) is thought to reside. Therefore, each unknown variable x_i is constrained by $\{x_i \in \Re \mid p_i \leq x_i \leq q_i\}$ (5) where p_i and q_i are the lower and upper bounds on x_i, respectively. Since the calculation of the solution cannot easily be performed using the intersection of the Q-box and S-ellipsoid, the Q-box is first approximated by a circumscribing ellipsoid, Q' (Figure 1c).

A new ellipsoid (P) now can be found that is a convex linear combination of the S-ellipsoid and Q'-ellipsoid (Figure 1d). This is the ellipsoid for which the new solution is computed. In addition to the double-sided inequality constraint places on the retrieved spectrum, an integral constraint is incorporated into the algorithm as well. This LaGrange Multiplier constraint is added to insure that the integrated reflectivity be equal through the deconvolution process.

The only other variable not accounted for is the turbulent intensity (w_σ). This is unknown as well and retrieval of w_σ is complicated by its location within the kernel itself. The best approach to find w_σ is to perform several retrieval iterations until errors in the spectrum are minimized. This approach works because removing too much or too little broadening will produce noticeable and predicable errors in the retrieved spectrum.

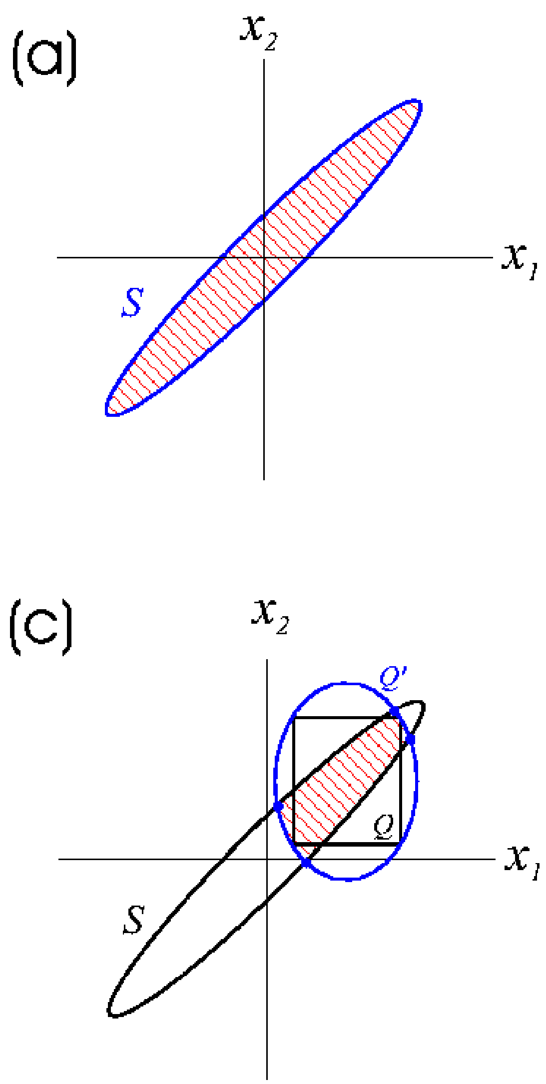

Figure 1. (a) In a 2-D example, S describes the a-level confidence ellipsoid for the solution x of an ill-posed problem. (b) However, we know that the true solution resides within the bounding box Q. (c) If the bounding box is parameterized using a circumscribing ellipsoid (Q'), a solution set with easily computed intersections is formed. (d) A new ellipsoid (P) is formed from the convex combination of Q' and S, and is where the new solution vector resides.

Examples

To illustrate this technique, we have chosen two spectra: a simulated Doppler spectrum from a liquid-phase stratus cloud and a real spectrum collected from a mixed-phase convective environment. The simulated spectrum is shown because the correct solution is known and comparison with the retrieved solution demonstrates the accuracy of the technique. Performing the deconvolution on the mixed-phase spectrum demonstrates the most difficult situation for interpretation of Doppler spectra.

The simulated spectrum is created by first using a drop-size distribution measured from an aircraft penetration of a stratus cloud. Using the diameter versus fall velocity function cited in Babb and Verlinde (1999), a quiet-air spectrum is obtained. This spectrum is then numerically convolved using Eqs. (1) and (4) with a known value of turbulence, and Gaussian distributed random noise is also added to the signal. The result is a broadened noisy spectrum similar to that which is observed (Figure 2a). When the deconvolution algorithm is run the result is seen to agree well with the original quiet air spectrum (Figure 2b). Also, the initial and retrieved drop distributions are compared (Figure 2c), showing excellent agreement.

To demonstrate the potential for using this method to interpret mixed-phase radar observations, spectra were collected from the ARM 35-GHz radar located at the central facility. The data used to illustrate the potential of this technique were collected from an incident where a stratus cloud existed in conjunction with convective cirrus outflow (Figure 3). The case shows the complexity that must be addressed if a long-term cloud parameter retrieval algorithm is to be developed. Furthermore, this case illustrates the potential for individual, detailed microphysical studies to be performed such as the effects of the interaction between the cirrus fall-streak and stratus cloud.

Several sets of spectra were collected from both the cirrus cloud and stratus cloud (Figure 4). These spectra were then processed using the constrained inversion algorithm to remove the spectral broadening by turbulence. The resulting quiet-air spectra show a great degree of detail including several modes. It is speculated that microphysical information can be directly retrieved (in the case of liquid phases clouds) or retrieved via parameterizations (for ice or mixed phase clouds) from these spectra. Unfortunately, actual spectral reflectivity values were not available as of this writing, so such hypotheses could be tested. Also, it should be mentioned that microphysical retrieval is highly dependent on the velocity resolution of the collected spectra. Therefore, in order to provide the most accurate retrievals this resolution should be maximized, by either increasing the FFT length or decreasing the Nyquist velocity of the spectra.

Conclusions

A method was developed to deconvolve turbulence broadening in radar Doppler spectra. This method is based on linear algebra techniques, which allow incorporation of a priori knowledge about the expected solution. These constraints help to control the often encountered problem of measurement and computational noise contamination of the retrieved signal. The algorithm used is adapted from a sub-optimal double-sided constraint technique presented in the mathematical literature.

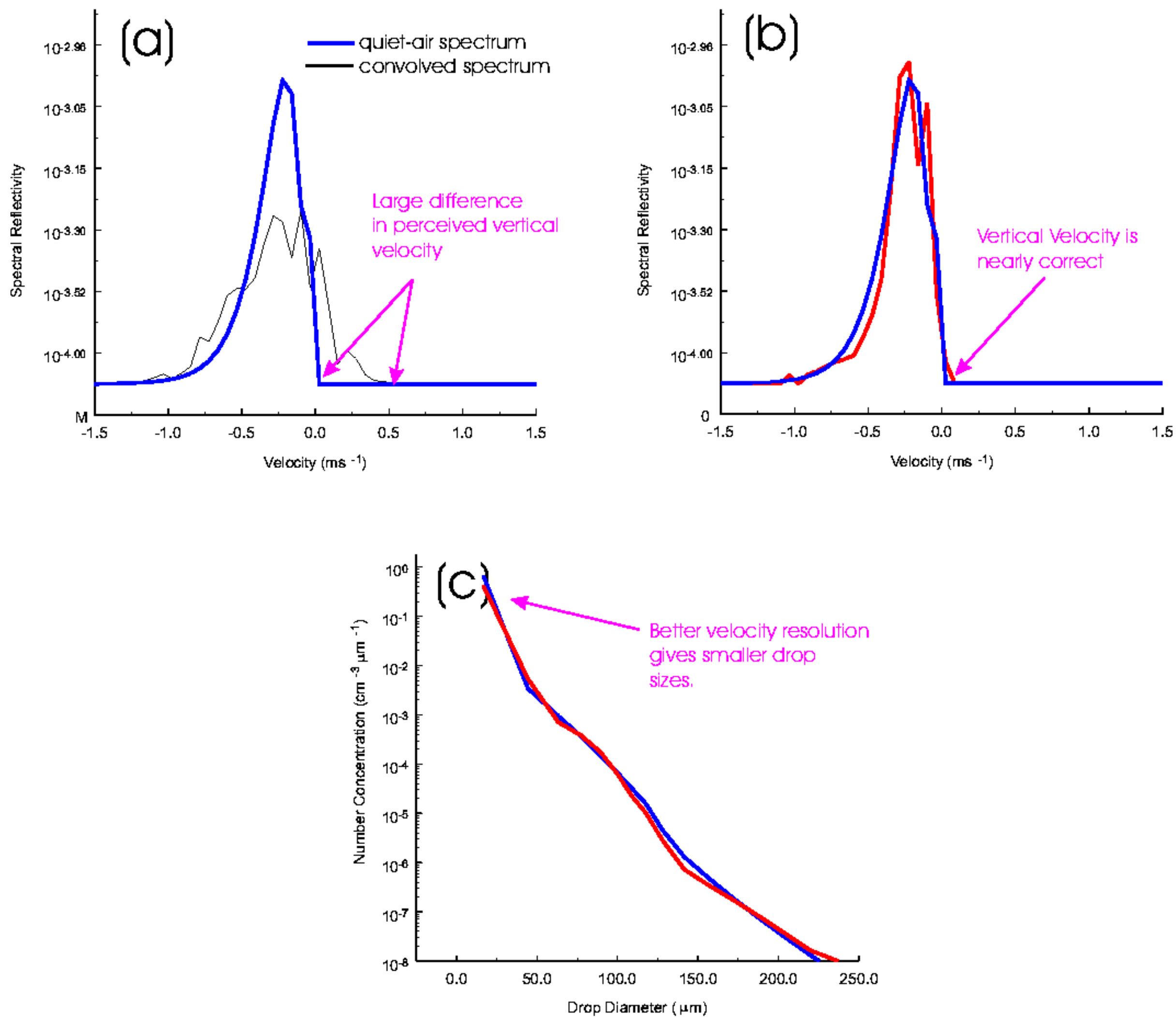

Figure 2. (a) A 64-point quiet-air spectrum is created from the drop distribution given in (c). The spectrum is then convolved with a gaussian filter, and noise is added. (b) Shows the sub-optimal retrieval of the convolved spectrum compared to the original quiet-air spectrum. (c) Shows a comparison between input and retrieved drop distributions.

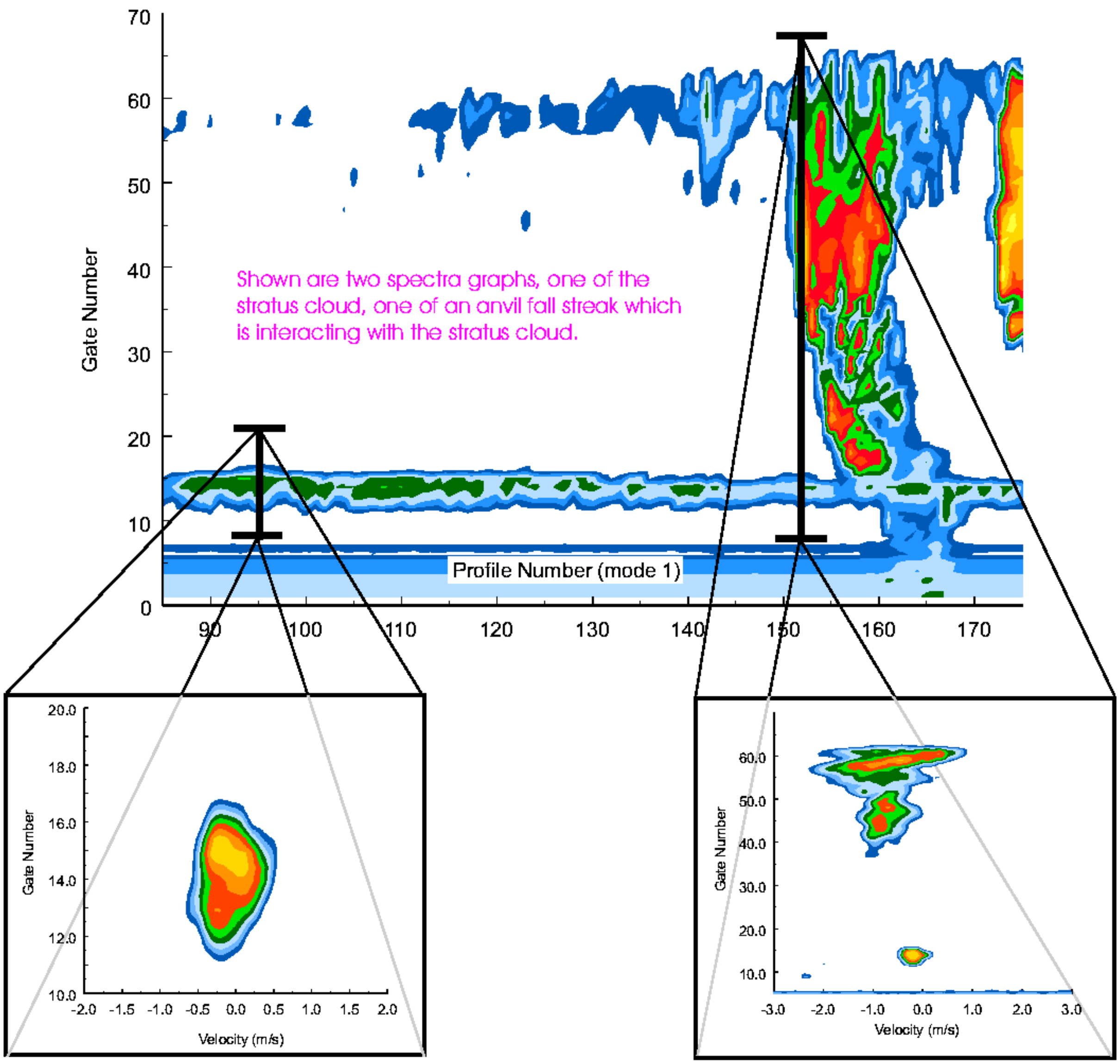

Figure 3. Time versus height plot of reflectivity showing a low-level stratus cloud and upper-level convective outflow. Two spectrographs are shown at the times indicated.

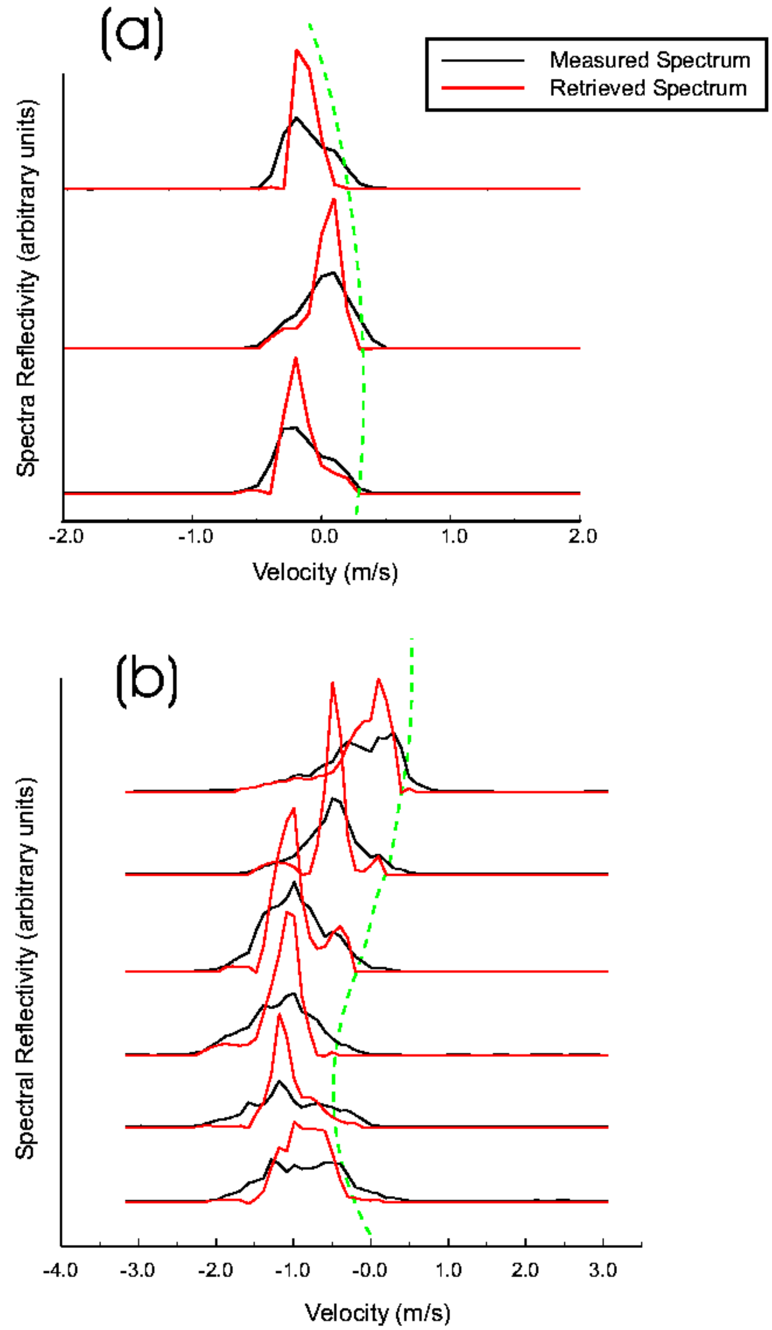

Figure 4. (a) Shown are the measured and retrieved spectra from a profile through the stratus cloud. We can see a definite bi-modal nature in both sets of spectra. Also shown (green dashed) is the volume-scale vertical velocity. (b) Shows the measured and retrieved spectra from a profile through the cirrus cloud. As above, the volume-scale vertical velocity is illustrated by the green dashed line.

Tests of the algorithm using both simulated and observed date produced excellent results. The advantage of this technique over other spectra-based retrieval algorithms is that no assumptions about the drop distribution (i.e., shape, phase, etc.) are needed to recover the quiet-air spectrum. Once this is accomplished, drop distributions are easily recovered via drop-size versus fall velocity and reflectivity versus size and number relationships. Ice and mixed-phase clouds present more of a challenge than liquid phase clouds; however, there is a large collection of work detailing fall velocity and reflectivity parameterizations for a wide variety of ice crystals (e.g., Aydin and Tang 1997).

Further testing is currently being conducted in order to determine the algorithm's robustness. Numerical simulations are being used to test the algorithm's response to known situations. In addition, retrievals are being conducted on a wide variety of observed spectra from different cloud types. Preliminary results indicate that the algorithm performs well for a majority of cases. In those instances where spectra are too weak or overly noisy, the algorithm dramatically fails. Efforts are being made to automate the entire retrieval process and provide a set of accuracy metrics with the retrieved spectrum.

Acknowledgments

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