Conceptually New Approach to Wind Measurements by Spaced Receiver Radars

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Introduction

Spaced receiver (SR) radars have shown clear advantages over conventional Doppler beam swinging radars for the measurements of wind profiles at high temporal and spatial resolution. Several effective techniques for wind and turbulence measurements have been developed (Holloway et al. 1997, Cohn et al. 1997). These techniques provide a solution to the diffraction problem by either directly assuming models of the diffraction pattern, or by assuming models of the refractive index field and relating those to the diffraction pattern. All the techniques consider the cross correlations and cross spectra of echoes in spaced receivers, which are the large-scale characteristics of the echoes. It is, however, well known that the large-scale characteristics of the refractive index field depend on atmospheric conditions, so no one model can be universal. This non-universality complicates application of the correlation-function-based techniques for operational, real-time wind and turbulence measurements.

We present a conceptually new approach to wind measurement by SR radars, which is not based on any specific model of the refractive index field or diffraction pattern. The approach uses universal features of the local, small-scale characteristics of the echoes. One can expect that such an approach could provide more universal, robust techniques for the wind measurements by SR radars.

An Empirical Model for the Local Structure of Echoes

Imagine an infinite number of receivers (R_i,i=1,2,...,∞) and transmitter T that illuminates the volume at range l with characteristic sizes L_{xy} and L_z (see Figure 1). The receiving fields of view are equal to or larger than the transmitting beam, which they are directed parallel to. The focuses of these receivers continuously cover a small volume V_s around the range l at the transmitter beam such that any

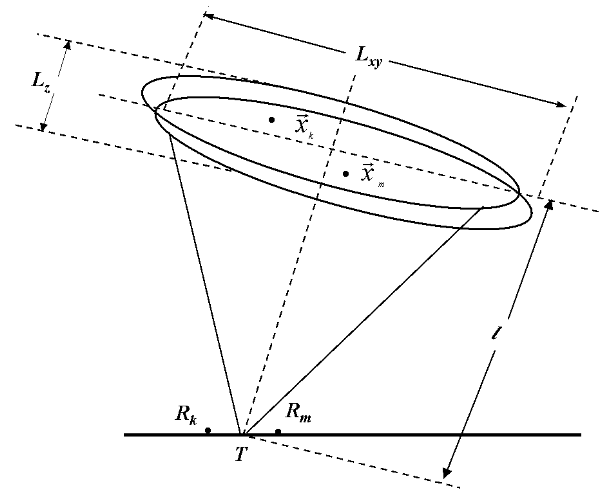

Figure 1. A schematic depicting transmitter T and two receivers R_k and R_m focused in two arbitrary locations $\vec{x}_k$ and $\vec{x}_m$ within the volume V_s.

two locations $\vec{x}_k$ and $\vec{x}_m$ from this volume satisfy the condition $|\vec{x}_k - \vec{x}_m| << L$ where L = min (l, L_{xy}, L_z). These receivers provide the field of echoes (FE) $s(\vec{x}, t), \vec{x} \in V_s$, as a function of time t from the refractive index irregularities within the illuminated volume.

One can ask the following questions:

- Does the FE $s(\vec{x}, t)$ have any universal features, i.e., features that are independent of the refractive index field structure?
- If they exist, can these features be formalized?

It is shown in this section that the local, small-scale characteristics of FE do have some universal features, and their description is presented.

An Empirical Model for the "Ideal" Echoes

Assume the receivers whose focuses cover the volume V_s in Figure 1 are identical, and their echoes contain no noise. The FE $S(\vec{x},t)$ from such receivers contains only atmospheric signal, and it will be called the "ideal" FE (IFE) from the "ideal" receivers. Characteristics of IFE are considered in this subsection. The IFE contains no contribution from noise or ground clutter.

The signal $S(\vec{x},t)$ from any location $\vec{x} \in V_s$ at instant t is an integrand over a huge volume of the order $(L_z L_{xy}^2)$; hence, this is the large-scale characteristic of IFE. Consider the local increments $\Delta S(\vec{x},\vec{r},t,\tau) = S(\vec{x},t) - S(\vec{x}+\vec{r},t+\tau)$. This operation is a filter that extracts variations of $S(\vec{x},t)$ with spatial and temporal scales $|\vec{r}|$ and τ. Consider the increments ΔS at very small separations $|\vec{r}|$ << L such as $\vec{x}+\vec{r} \in V_s$ for any $\vec{x} \in V_s$. Consider further the increments at very small τ << T where T is the minimal from all the external time scales, or, more rigorously, at $\tau \to 0$.

The increments $\Delta S(\vec{x},\vec{r},t,\tau)$ at $\vec{r} << L$ and $\tau \to 0$ characterize the local, small-scale structure of the IFE ($S(\vec{x},t)$). Below, we will consider the functional behavior of the mean square values of such increments $\left\langle \Delta S^2(\vec{x},\vec{r},t+\tau)\right\rangle$. Following a terminology of the turbulence theory, these values will be called the second-order structure functions of the IFE. The standard notation

$$D(\vec{x}_k,\vec{x}_m,t,\tau) = \left\langle [S(\vec{x}_k,t) - S(\vec{x}_m,t+\tau)]^2 \right\rangle$$

will be used. Let us formulate the basic assumptions about the IFE.

Assumption 1. There should exist such the minimal time T_{st} that for any interval T_{av} £ T_{st} the IFE can be considered statistically stationary. It follows from this assumption that one can estimate any statistical characteristic of $S(\vec{x},t)$ as a time average over an interval $T_{av} \le T_s$. We will consider the IFE within a small volume V_s during a fixed time interval $[t - T_{av}, t]$. All time averages over this interval do not depend on t.

Assumption 2. The IFE is locally statistically homogeneous at $\vec{x} \in V_s$. It follows from this assumption that $\left\langle S^2(\vec{x},t)\right\rangle$ is constant, and $D(\vec{x}_k,\vec{x}_m,t,\tau)$ depends only on $\vec{x}_k - \vec{x}_m$ and τ for any $\vec{x} \in V_s$, $\vec{x}_k \in V_s$, and $\vec{x}_m \in V_s$.

Assumption 3. The IFE is smooth in both space and time. More rigorously, we assume that $S(\vec{x},t)$ belongs to the class $C^{(2)}_{\vec{x},t}$. It follows from this assumption that one can decompose $S(\vec{x},t)$ into Taylor series in the vicinity of $\vec{x}$ and t.

Assumption 4. There should exist such the minimal time T_{ad} £ T_{av} that all components of the mean advection speed of scatterers in the illuminated volume can be considered approximately equal to the respective components of the mean wind speed. This assumption is usually taken for granted in remote sensing.

Consider three arbitrary locations: $\vec{x}_k \in V_s$, $\vec{x}_m \in V_s$, and $\vec{x}_n \in V_s$. In other words, consider three "ideal" receivers focused in these three locations. One can write for any statistically stationary and locally homogeneous random field that

$$\begin{aligned} D(\vec{x}_k,\vec{x}_m,t,\tau) &= D(\vec{x}_k,\vec{x}_n,t,0) \\ &+ 2\left\langle S(\vec{x}_k,t)[S(\vec{x}_n,t) - S(\vec{x}_m,t+\tau)]\right\rangle \end{aligned} \qquad (1)$$

The first RHS term in Eq. (1) is the spatial structure function at τ = 0 for the locations $\vec{x}_k$ and $\vec{x}_n$, and the second RHS term describes a correlation between the large- and the small-scale characteristics of the IFE.

Consider $\vec{x}_n$ to be located $-\vec{u}\tau$ downwind from $\vec{x}_m$ where $\vec{u}$ is the mean wind speed, i.e., $\Delta\vec{x}_{nm} = -\vec{u}\tau$. Hereafter, $\Delta\vec{x}_{ij} = \vec{x}_i - \vec{x}_j$.

Let us estimate the first RHS term in Eq. (1). One can write:

$$\begin{aligned} S(\vec{x}_n,t) &= S(\vec{x}_k,t) \\ &+ \frac{\nabla S(\vec{x}_k,t)\bullet\Delta\vec{x}_{nk}}{|\Delta\vec{x}_{nk}|}|\Delta\vec{x}_{nk}| + 0\left(|\Delta\vec{x}_{nk}|^2\right) \end{aligned}$$

that gives:

$$\begin{aligned} D(\vec{x}_k,\vec{x}_n,t,0) &= b(\vec{x}_k,\vec{x}_n)\,[\Delta\vec{x}_{mk}\bullet\Delta\vec{x}_{mk} \\ &- 2\tau\Delta\vec{x}_{mk}\bullet\vec{u}] + 0\left(\tau^2, |\Delta\vec{x}_{mk}|^3\right) \end{aligned} \qquad (2)$$

where the constant

$$b(\vec{x}_k,\vec{x}_n)=\left\langle\left[\frac{\Delta S(\vec{x}_k,t)\bullet\Delta\vec{x}_{nk}}{|\Delta\vec{x}_{nk}|}\right]^2\right\rangle$$

will be defined later.

Consider the second RHS term in Eq. (1). It can be shown that at $\tau\to 0$,

$$\left\langle S(\vec{x}_k,t)[S(\vec{x}_n,t)-S(\vec{x}_m,t+\tau)]\right\rangle=0(\tau^2) \tag{3}$$

One can combine Eqs. (1) through (3) at $\tau\to 0$ as:

$$\begin{aligned}&D(\vec{x}_k,\vec{x}_m,t,\tau)=\\&\quad b(\vec{x}_k,\vec{x}_n)\left[\Delta\vec{x}_{mk}\bullet\Delta\vec{x}_{mk}-2\tau\Delta\vec{x}_{mk}\bullet\vec{u}\right]\end{aligned} \tag{4}$$

where $b(\vec{x}_k,\vec{x}_n)$ is the only unknown value. The time separation τ is a parameter in this equation, i.e., the equation can be applied to any $\tau\to 0$. In particular, at $\tau = 0$ one can write:

$$D(\vec{x}_k,\vec{x}_m,t,0)=b(\vec{x}_k,\vec{x}_n)\,\Delta\vec{x}_{mk}\bullet\Delta\vec{x}_{mk},$$

that gives:

$$b(\vec{x}_k,\vec{x}_n)=\frac{D(\vec{x}_k,\vec{x}_m,t,0)}{\Delta\vec{x}_{km}\bullet\Delta\vec{x}_{km}} \tag{5}$$

One can obtain from Eqs. (4) and (5) that at $\tau\to 0$

$$\frac{D(\vec{x}_k,\vec{x}_m,t,\tau)}{D(\vec{x}_k,\vec{x}_m,t,0)}=1-2\tau\frac{\Delta\vec{x}_{mk}\bullet\vec{u}}{\Delta\vec{x}_{mk}\bullet\Delta\vec{x}_{mk}} \tag{6}$$

This is the final formulation of a universal empirical model for the local structure of the IFE. The model relates the two-point second-order structure function of the IFE to the projection of the wind vector on the direction $\Delta\vec{x}_{mk}$.

An Empirical Model for Real Receivers

Consider now real echoes from real receivers. For simplicity of explanation, the gains of the receivers are assumed equalized. One can present a real echo $s(\vec{x}_i,t)$ for any $\vec{x}_i\in V_s$ as:

$$s(\vec{x}_i,t)=S(\vec{x}_i,t)+n(\vec{x}_i,t)+c(\vec{x}_i,t), \tag{7}$$

where $S(\vec{x}_i,t)$ is the “ideal” atmospheric echo, $n(\vec{x}_i,t)$ is the noise component with zero correlation time, e.g., thermal noise, and $c(\vec{x}_i,t)$ is the noise component with a non-zero correlation time, e.g., from hard targets and ground clutter.

Assumption 5. Atmospheric signal $S(\vec{x}_i,t)$ and noise components are not correlated, and neither are $n(\vec{x}_i,t)$ and $c(\vec{x}_i,t)$. It follows from this assumption that for any two locations $\vec{x}_k\in V_s$ and $\vec{x}_m\in V_s$ one can write:

$$\begin{aligned}&D_s(\vec{x}_k,\vec{x}_m,t,\tau)=D(\vec{x}_k,\vec{x}_m,t,\tau)\\&+D_n(\vec{x}_k,\vec{x}_m,t,\tau)+D_c(\vec{x}_k,\vec{x}_m,t,\tau)\end{aligned} \tag{8}$$

Here, $D_q(\vec{x}_k,\vec{x}_m,t,\tau)=\left\langle[q(\vec{x}_k,t)-q(\vec{x}_m,t+\tau)]^2\right\rangle$ for q = s, n, or c. Using assumptions 1 through 3, one can obtain:

$$\begin{aligned}D_n(\vec{x}_k,\vec{x}_m,t,\tau)&=\left\langle n^2(\vec{x}_k,t)\right\rangle\\&\quad+\left\langle n^2(\vec{x}_m,t)\right\rangle\ \text{at any }\tau,\\D_c(\vec{x}_k,\vec{x}_m,t,\tau)&=0\ \text{ at }\tau\to 0\end{aligned}$$

that gives:

$$\begin{aligned}&D(\vec{x}_k,\vec{x}_m,t,\tau)=D_s(\vec{x}_k,\vec{x}_m,t,\tau)\\&\quad-\left\langle n^2(\vec{x}_k,t)\right\rangle-\left\langle n^2(\vec{x}_m,t)\right\rangle\end{aligned} \tag{9}$$

Eq. (9) contains unknown values $\left\langle n^2(\vec{x}_k,t)\right\rangle$ and $\left\langle n^2(\vec{x}_m,t)\right\rangle$. Using assumption 5, one can write for any location $\vec{x}_i\in V_s$ at any $\tau\neq 0$ that:

$$D_s(\vec{x}_i,\vec{x}_i,t,\tau)=D(\vec{x}_i,\vec{x}_i,t,\tau)+2\left\langle n^2(\vec{x}_i,t)\right\rangle$$

Using Taylor's decomposition, one can write:

$$S(\vec{x}_i,t+\tau)=S(\vec{x}_i,t)+\frac{\partial S(\vec{x}_i,t)}{\partial t}\tau+0(\tau^2)$$

Combining these equations, one obtains at $\tau\to 0$.

$$D_s(\vec{x}_i,\vec{x}_i,t,\tau)=2\left\langle n^2(\vec{x}_i,t)\right\rangle+\beta(\vec{x}_i)\tau^2 \tag{10}$$

where $\beta(\vec{x}_i) = \left\langle [\partial S(\vec{x}_i, t)/\partial t]^2 \right\rangle$ is an irrelevant constant. One can now write the final formulation of an empirical model for the local structure of the real FE as:

$$\frac{D_s(\vec{x}_k, \vec{x}_m, t, \tau) - \left\langle n^2(\vec{x}_k, t)\right\rangle - \left\langle n^2(\vec{x}_m, t)\right\rangle}{D_s(\vec{x}_k, \vec{x}_m, t, 0) - \left\langle n^2(\vec{x}_k, t)\right\rangle - \left\langle n^2(\vec{x}_m, t)\right\rangle} = 1 - 2\pi \frac{\Delta\vec{x}_{mk} \bullet \vec{u}}{\Delta\vec{x}_{mk} \bullet \Delta\vec{x}_{mk}} \quad (11)$$

where $\left\langle n^2(\vec{x}, t)\right\rangle$ and $\left\langle n^2(\vec{x}_m, t)\right\rangle$ are defined from Eq.(10) at $\tau \to 0$.

Eq. (11) reveals a remarkable feature of the model: it is not significantly affected by the noise component with non-zero correlation time. In particular, by ground clutter and hard targets.

The model provides simple operational algorithms for real-time, high-resolution measurements of the projection of the wind vector on the direction $\vec{x}_m - \vec{x}_k$ by SR radars.

Operational Algorithm for MAPR

The National Center for Atmospheric Research's (NCAR's) multiple antenna profiler (MAPR) is a modified version of the commercially available Radian LAP-3000 915-MHz boundary layer profiler; for detailed descriptions see Cohn et al. (1997) and references therein. MAPR is a spaced antenna system with the full (four-panel) antenna used to transmit with each single antenna panel receiving backscatted energy processed through four independent receiver channels (Figure 2). Thus, there are four antennas and six antenna-pair baselines for estimating the projections of the wind vector from Eqs. (11) and (10). One can choose the MAPR-based coordinate system such as u_x wind component is directed, e.g., along the baseline (0,1), and u_y component is directed, e.g., along the baseline (0,3), and then rewrite Eq. (11) for any specified pair of antennas. For example:

$$\frac{D_s(\vec{x}_0, \vec{x}_2, t, \tau) - \left\langle n^2(\vec{x}_0, t)\right\rangle - \left\langle n^2(\vec{x}_2, t)\right\rangle}{D_s(\vec{x}_0, \vec{x}_2, t, 0) - \left\langle n^2(\vec{x}_0, t)\right\rangle - \left\langle n^2(\vec{x}_2, t)\right\rangle} = 1 - \frac{\tau}{\delta}(\vec{u}_x + \vec{u}_y),$$

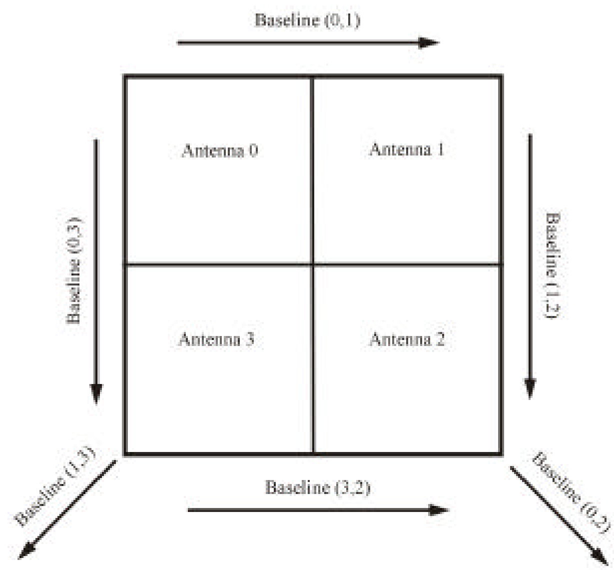

Figure 2. NCAR's MAPR antenna configuration.

Here, $\vec{x}_i, i = 0, 1$ denotes the i-th antenna, and δ denotes an effective distance between the antennas.

The above equations were used to estimate the horizontal wind components u_x and u_y for the experiment at the National Oceanic and Atmospheric Administration (NOAA) Boulder Atmospheric Observatory (BAO); see Cohn et al. (1997) for experimental details. As an illustration, the horizontal wind speed $V_h = \sqrt{u_x^2 + u_y^2}$ at 30s resolution is shown in Figure 3. The presented results were obtained on May 17, 1995, at 300 m above ground. Also shown are measurements of V_h by the sonic anemometer that was located on a tower approximately 600 m apart from the MAPR. Most of time, the values of V_h from the presented algorithm are in reasonable agreement with those from sonic anemometer. However, there is no agreement at approximately 1:30 - 2:00 UT and 11:30 - 13:00 UT. During these periods of time, the signal-to-noise ratio (SNR) was too low to provide reliable data. The results in Figure 3 demonstrate that the empirical model [Eqs. (11), (10)] adequately describes the local structure of the field of echoes, and the model-based algorithms show a reasonable skill for wind measurements by SR radars.

Summary

A universal empirical model for local structure of the field of echoes from SR radars has been developed. It is based on

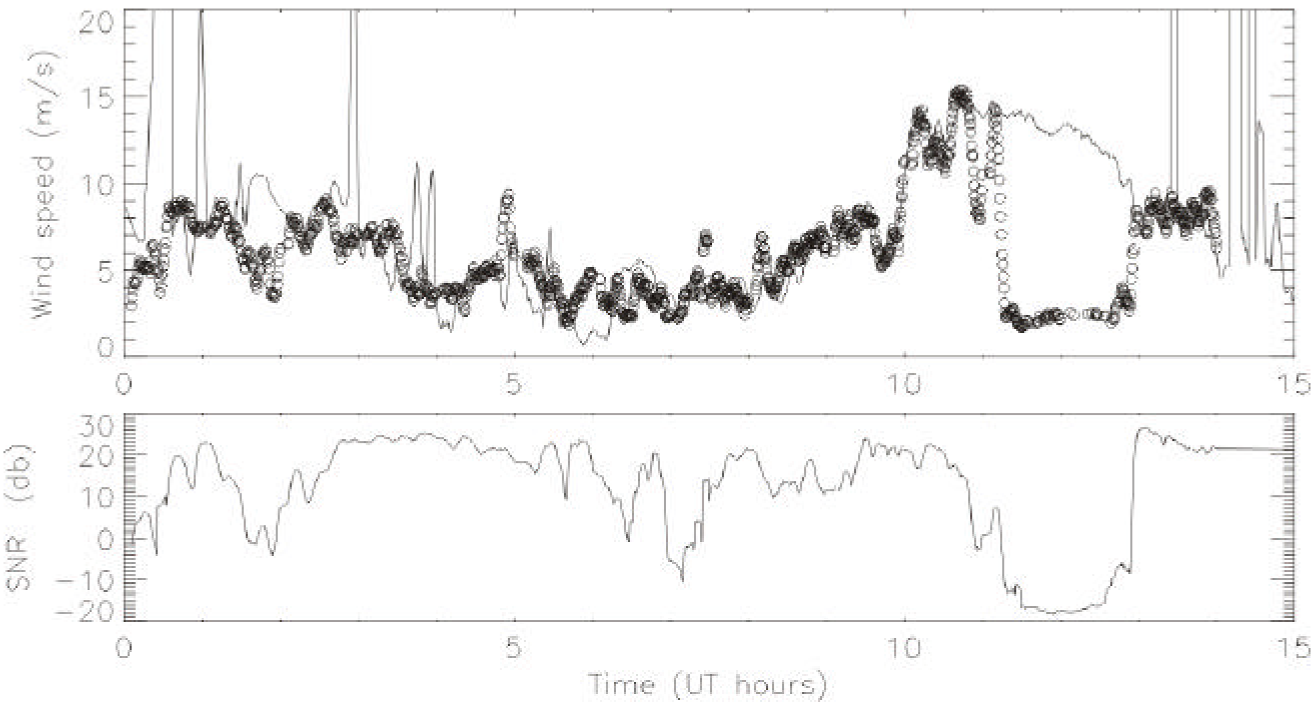

Figure 3. Upper panel: Horizontal wind speed at 30s resolution measured with MAPR (circles) and with the sonic anemometer (solid line). Lower panel: the SNR for the MAPR signals.

only five assumptions of a general nature. These assumptions are also used in all existing models, and usually in much stronger formulations.

The model is conceptually new in three aspects: 1) it is universal because its derivation is not based on any specific parameterization of refractive index field or diffraction pattern; 2) it has been derived by using qualitative physical analysis rather than a solution to the diffraction problem; and 3) it describes the local, small-scale characteristics of the field of echoes by using the structure functions rather than the large-scale characteristics described by the correlation functions.

The model is exact, i.e., it does not contain any empirical function and/or constant to be defined.

The model provides simple operational algorithms for real-time wind measurements by SR radars. The effectiveness of the model-based algorithm for wind measurements by MAPR has been demonstrated.

Acknowledgments

NCAR is sponsored by the National Science Foundation. This work was sponsored in part by the grant DE-A105-90ER61070 from the Atmospheric Radiation Measurement (ARM) Program, and in part by the NCAR Director's fund.

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