Prescribing Advection in Single-Column Models

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Introduction

Both single-column models (SCMs) and cloud ensemble models (CEMs) are often forced with observed, objectively analyzed fields (Randall et al. 1996). Consider an arbitrary scalar variable, q, satisfying a flux-form conservation equation:

$$\frac{\partial q}{\partial t} = -\nabla \bullet (Vq) - \frac{\partial}{\partial p}(\omega q) + P \tag{1}$$

Here P represents the "physics" that affects q. The corresponding continuity equation is

$$\nabla \bullet V + \frac{\partial \omega}{\partial p} = 0 \tag{2}$$

By using Equation 2, we can rewrite Equation 1 in the "advective" form:

$$\frac{\partial q}{\partial t} = -V \bullet \nabla q - \omega \frac{\partial q}{\partial p} + P. \tag{3}$$

A one-dimensional (1D) model cannot predict the large-scale divergence, so if Equation 2 is to be used to obtain the vertical velocity, then we must prescribe $\nabla \bullet V$. Similarly, a 1D model cannot determine $-V \bullet (\nabla q)$ or $-V \bullet \nabla q$, so we need to prescribe some information about the horizontal advection of q.

Some investigators have experimented with an artificial "relaxation" term added to the right-hand side of Equation 3, i.e.,

$$\frac{\partial q}{\partial t} = -V \bullet \nabla q - \omega \frac{\partial q}{\partial p} + P + \frac{(qobs - q)}{\tau}, \tag{4}$$

where q_{obs} is the observed value of q, and τ is a specified "relaxation time scale," which is specified to be on the order of a day to half a day. A problem with the relaxation term is that it does not represent any real physical process.

Consider three different methods to include the advection terms in an SCM or CEM:

Revealed Forcing

One possibility is simply to compute directly from the observations, and then prescribe these values in the 1D model, integrating Equation 3. With this simple approach, errors in the predicted vertical distribution of q have no effect on the advective tendency of q.

Vertical Advective Forcing

A simple modification of the above approach consists of prescribing and ω from the observations, and using the *predicted* profile of q, together with the prescribed ω, to evaluate $-\omega \frac{\partial q}{\partial p}$ as the model runs.

Vertical Flux Forcing

Many large-scale models, especially finite-difference models, use the flux form, Equation 1, to predict q. It is possible to retain the flux form for vertical advection in a 1D model, as follows. Splitting the horizontal advection term of Equation 1 into two pieces gives

$$\frac{\partial q}{\partial t} = -V \bullet \nabla q - q \nabla \bullet V - \frac{\partial}{\partial p}(\omega q) + P. \tag{5}$$

We can prescribe $V \bullet \nabla q$ and $\nabla \bullet V$ from observations. By integrating the continuity equation, Equation 2, we can obtain $\omega(p)$from $\nabla \bullet V$. Then Equation 5 can be used to predict q.

Suppose that q represents the mixing ratio of water vapor. The total lateral moisture flux convergence is then given by $-\nabla \bullet Vq - qV \bullet \nabla$. The first term, $-\nabla \bullet Vq$, is prescribed. The second depends on both the prescribed wind convergence, $-V \bullet \nabla$,and the simulated vapor mixing ratio, q. From

$$\frac{\partial q}{\partial t} \sim -q\nabla \bullet V \qquad (6)$$

it is apparent that q will tend to grow exponentially (and catastrophically) in a layer that has $\nabla \bullet V < 0$, and to decay exponentially in a layer that has $\nabla \bullet V > 0$.

Relaxation Forcing

Using Gauss's Theorem, we can rewrite the horizontal flux divergence term of Equation 1 as

$$\nabla \bullet (Vq) = \frac{1}{A}\left[-(V_{in}\Delta l_{in} q_{in}) + (V_{out}\Delta l_{out} q_{out})\right], \qquad (7)$$

where the first term represents the inflow, and the second represents the outflow. Next, we modify Equation 7 by adding and subtracting terms involving q:

$$\nabla \bullet (Vq) = \frac{1}{A}\left\{\left[V_{in}\Delta l_{in}(q_{in} - q)\right] + \left[V_{out}\Delta l_{out}(q_{out} - q)\right]\right\} + \frac{q}{A}\left[V_{in}\Delta l_{in}\right] + \left[V_{out}\Delta l_{out}\right] \qquad (8)$$

We recognize the quantity on the second line of the right-hand side of Equation 8 as $q\nabla \bullet V$, so that Equation 8 is equivalent to

$$V \bullet \nabla q = \frac{1}{A}\left\{-[V_{in}\Delta l_{in}(q_{in} - q)] + [V_{out}\Delta l_{out}(q_{out} - q)]\right\} \qquad (9)$$

Now suppose that

$$q - q_{out} = f(q_{in} - q) \qquad (10)$$

With the use of Equation 10, we can re-write Equation 9 as

$$-V \bullet \nabla q = \frac{(q_{in} - q)}{\tau_{adv}} \qquad (11)$$

where we define

$$\frac{1}{\tau_{adv}} = \frac{(V_{in}\Delta l_{in}) + f(V_{out}\Delta l_{out})}{A} \qquad (12)$$

Finally, we substitute Equation 11 into Equation 3 to obtain

$$\frac{\partial q}{\partial t} = \frac{(q_{in} - q)}{\tau_{adv}} - \omega\frac{\partial q}{\partial p} + P \qquad (13)$$

The meaning of Equation 11 and Equation 13 is that horizontal advection acts like a relaxation of q towards q_{in}, with relaxation time scale τ_{adv}.

When we directly insert the observed value of $-V \bullet \nabla$ into Equation 3, errors in the prescribed horizontal advective tendency and/or errors in the SCM physics can drive the simulated sounding away from the evolving observed sounding; the model "gets lost." Because the inserted data do not contain information about the actual value of q, the model is not able to find its way back home.

Compare Equation 13 with Equation 4. The relaxation term of Equation 4 is added artificially, *in addition* to the horizontal advection term. The relaxation time scale in Equation 4 has to be arbitrarily specified. The relaxation in Equation 4 is towards q_{obs}, the observed value of q in the region. The relaxation term of Equation 4 cannot be compared with observations because it does not represent a real physical process. In contrast, the relaxation term of Equation 13 *is identically the horizontal advection term*. The relaxation time scale τ_{adv} can be computed directly from the data and does not have to be specified arbitrarily. The relaxation in Equation 13 is towards q_{in}, the observed properties of the air entering the region. The relaxation term of Equation 13 can be compared with the objectively analyzed value of $-V \bullet \nabla q$.

Before we can actually use Equation 13, it is necessary to diagnose q_{in} and τ_{adv} from the objective analysis scheme. With some simplifying assumptions we can write

$$\tau_{adv} = \frac{d}{2V} \qquad (14)$$

and Equation 11 yields

$$q_{in} = q - \tau_{adv} V \bullet \nabla q \qquad (15)$$

All of the quantities on the right-hand sides of Equation 14 and Equation 15 are observable.

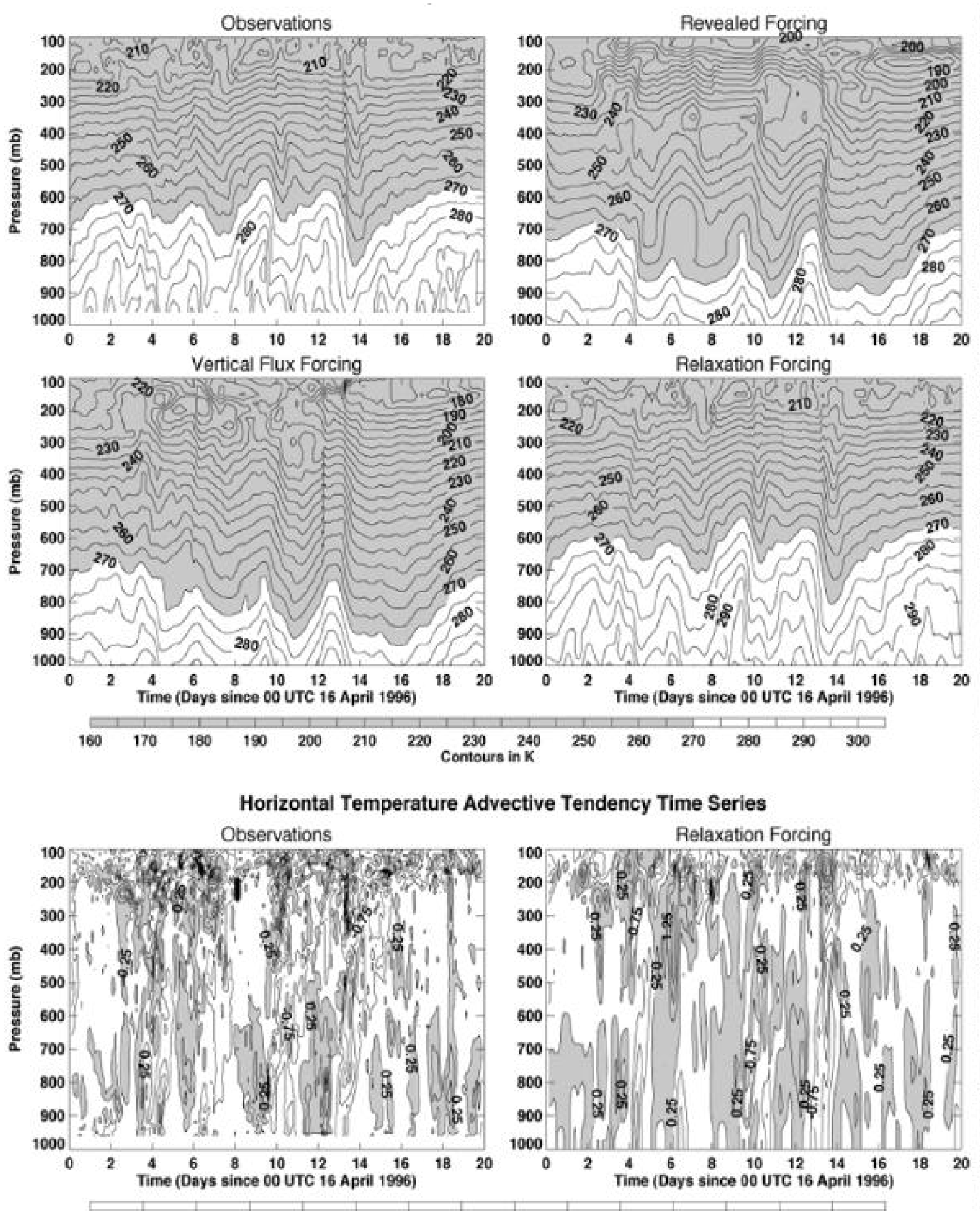

Figure 1. Time-height sequence of temperature for the April 1996 IOP: (top left) observed; (top right) simulated using revealed forcing; (center left) simulated using vertical flux forcing; (center right) simulated using relaxation forcing. Also, time-height sequence of temperature tendency due to horizontal advection for: (bottom left) observed; (bottom right) simulated using relaxation forcing.

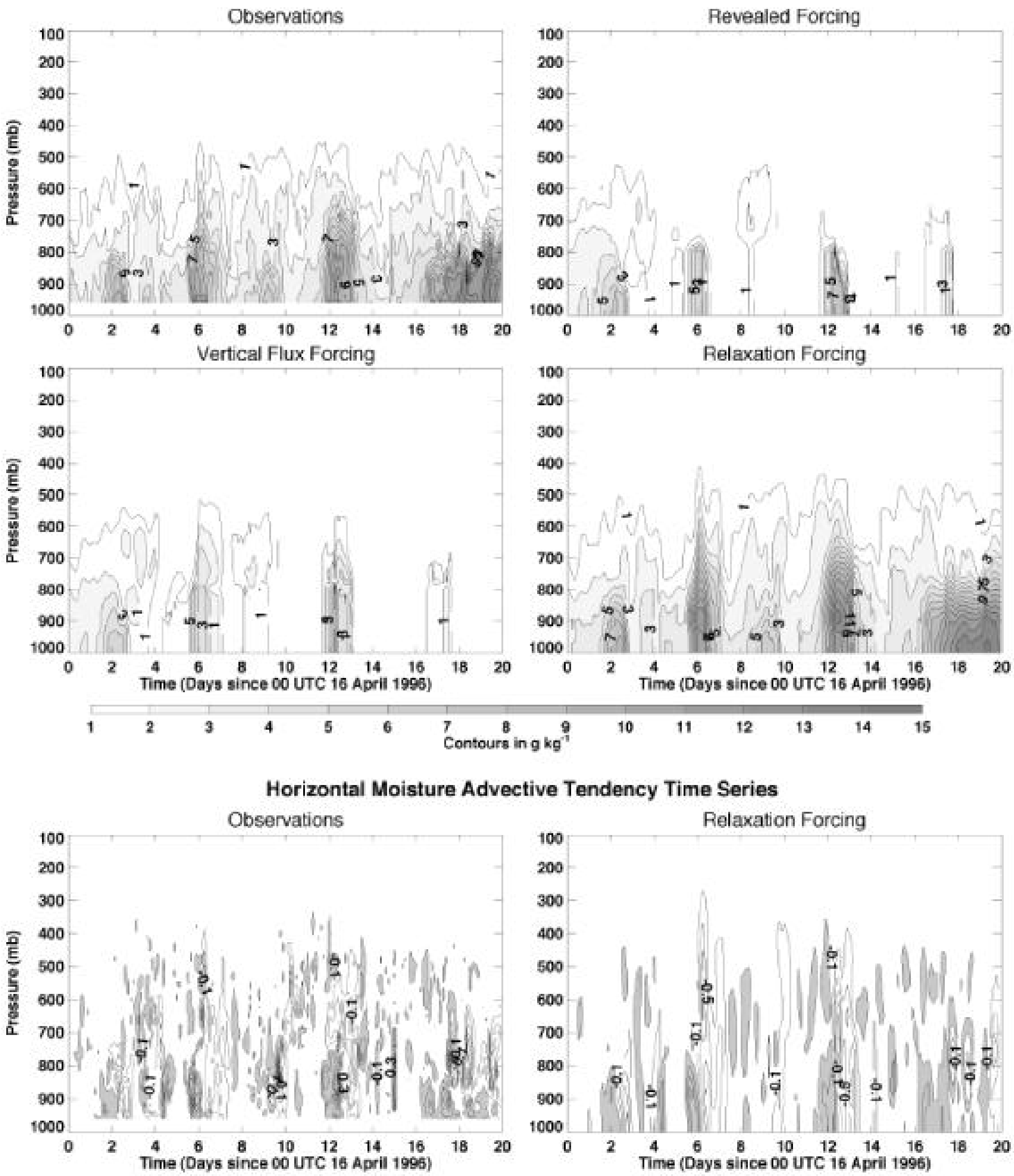

Figure 2. As in Figure 1, but for water vapor mixing ratio.

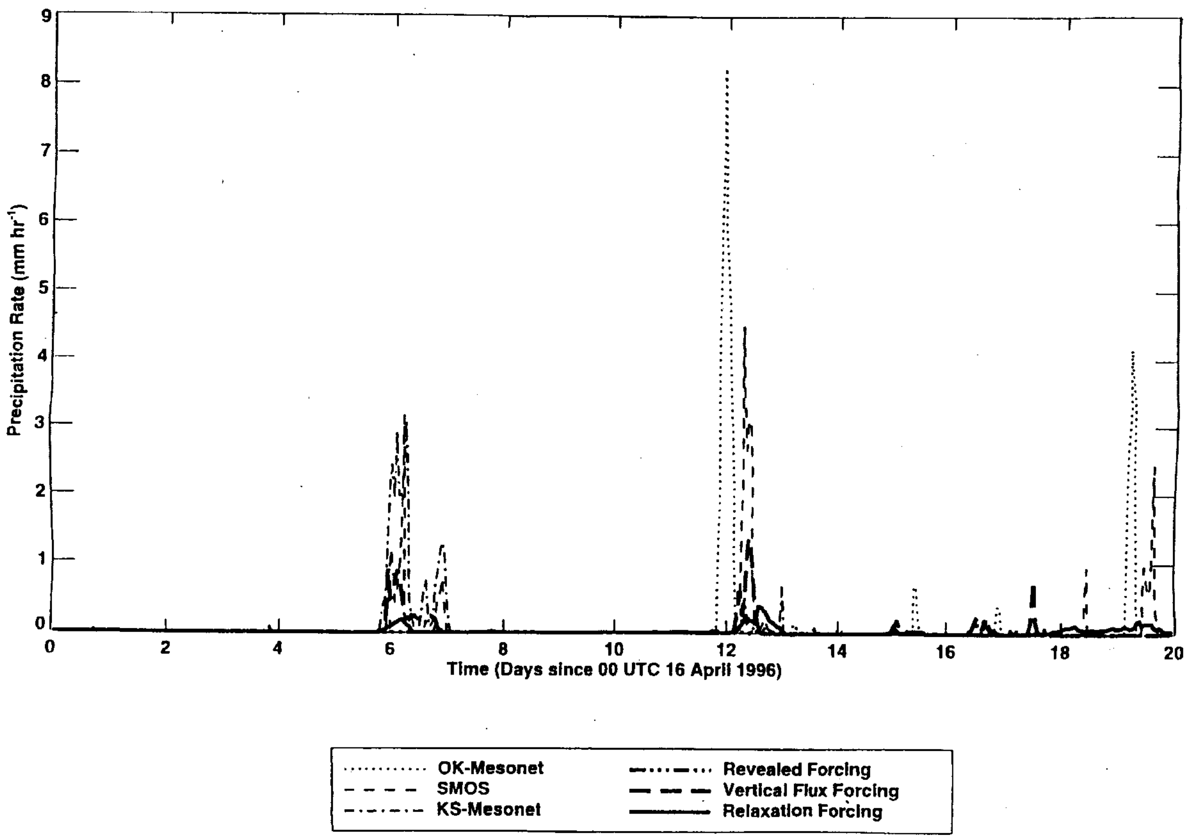

Figure 3. Time sequence of simulated and observed precipitation rate for the April 1996 IOP.

References

Randall D. A., K.-M. Xu, R.J.C. Somerville, and S. Iacobellis, 1996: Single-column models and cloud ensemble models as links between observations and climate models. *J. of Climate*, **9**, 1683-1697.