Multifractal Modeling and Analysis of Radiation in Clouds: 5000 km to 50 cm

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Introduction

Existing general circulation models rely heavily on strong cloud homogeneity assumptions; the solar radiation is assumed to interact with highly unrealistic plane parallel (horizontally homogenous) clouds. In contrast, real clouds exhibit fractal structures and multifractal statistics over wides ranges of scale. For example, in Lovejoy et al. (1993) and Tessier et al. (1993), we showed that the isotropic energy spectrum of clouds is remarkably accurately scaling over the entire range of at least several hundred meters to several thousand kilometers. Since spectral analysis is a very sensitive indicator of scale invariance, the remarkably good scaling—even on individual cloud pictures —contradicts the standard model of the atmosphere, which is based on a hypothetical (and theoretically doubtful) "meso-scale gap" ("dimensional transition") separating two dimensional isotropic turbulence at large scales and three dimensional isotropic turbulence at small scales. However, because the dynamical velocity field is strongly nonlinearly coupled with the cloud field, such scaling was in fact predicted by the "unified scaling model" of the atmosphere (based on scaling stratification [Schertzer and Lovejoy 1985], [Lovejoy and Schertzer 1985, 1986]) and gives it strong support.

Because scaling cloud radiances imply strong (power law) resolution dependences in radiation measurements, any estimates of the earth's radiation budget require a good knowledge of both the limits and types of scaling, hence such knowledge is fundamental to the Atmospheric Radiation Measurement (ARM) Program. It is therefore important not only to better characterize the scaling in the range greater than 1 km (i.e., the range accessible to meteorological satellites), but also to discover the inner scale of the scaling. Below the (apparently nonexistent) mesoscale gap, the only other theoretically predicted fundamental length scale in clouds is the dissipation scale (ª1-10 mm). It is therefore important to investigate the scaling down to this small scale; i.e., to extend the range of scaling studies into the 1 km to 1 cm range.

This is the object of the first part of this paper, which uses hundreds of geostationary, polar orbiting, and ground-based images to investigate the scaling of cloud liquid water and visible and infrared radiances over the range ª5000 km to ª50 cm. The second part exploits the scaling to study multifractal scattering statistics (identifying a multifractal mechanism for anomalous absorption) and to create multifractal cloud simulations (including the crucial scaling of the vertical anisotropy, which is not included in other scaling models) to produce realistic radiation fields. It should be noted that the strong cloud variability found here is incompatible with the monofractal "bounded cascaded" (Cahalan 1994) or other weakly varying cloud models. It is therefore not surprising that we find that multifractal cloud variability gives a much larger bias (with respect to plane parallel assumptions) than those reported by other studies.

Multifractal Cloud Analysis and Modeling Over a Wide Range of Scales and Intensities

Testing Scaling Over a Wide Range of Scales - Cloud Liquid Water Data

One of our key objectives was to use ARM cloud liquid water content (LWC) estimates to simulate radiance fields using multifractal models. Although the ARM LWC data were acquired early in 1996, as of this writing, they still are not available. We therefore examined the range and type of scaling using King probe data from the First ISCCP[(a)] Regional Experiment (FIRE) experiment. The data set analyzed here is from various aircraft runs, with the longest containing 65,536 points (corresponding to a range of scales 5 m - 328 km).

Figures 1a and 1b show that all five LWC data sets are scaling over the whole range (energy spectrum $E(k)$ªk^{-b}), corresponding to a range of spatial scales from 5 m-330 km.

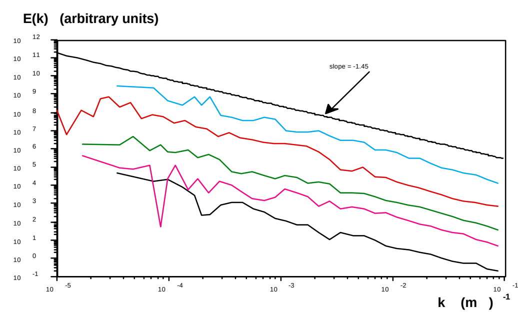

Figure 1a. Power spectrum of the five different data sets (averaged over 10 equally logarithmically spaced points on the k-axis and vertically offset). The absolute slopes with b = 1.45 is indicated (straight line on top of graph) for reference. The number of sets used to compute the average from top to bottom: 4, 3, 1, 2, 5. A constant aircraft speed of 100 m/s has been assumed.

Figure 1b is the longest single scaling spectrum of any geophysical quantity of which we are aware; the scaling is excellent, especially when it is considered that scaling is only a statistical symmetry; a priori, we expect large fluctuations (intermittent spectral spikes) on single realizations such as this. In Lovejoy and Schertzer (1995) we discuss this, as well as multifractal parameter estimates for LWC. A specific problem noted was the difficulty in accurately measuring the

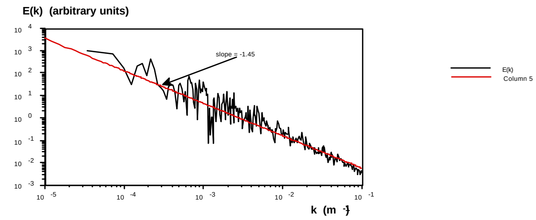

Figure 1b. Ensemble average power spectrum (averaged over 100 equally logarithmically spaced points per magnitude on the k axis). The ensemble average of the squared moduli of the 15 equal sized data sets yields a power spectrum with a spectral slope bÅ1.45.

frequency of occurrence of low-density cloud regions. The scattering theory discussed below indicates that these "holes" may play a crucial role in radiative transfer, so that more work is needed to characterize the LWC statistics (i.e., difficulty in measuring the index of multifractalilty a, for LWC; see below).

Extensions to Larger and Smaller Scales and Larger Quantities of Data

To get more precise estimates of the type and range of cloud scaling, we have begun a large-scale analysis of satellite cloud data. Figure 2a shows the result of averaging spectra from 29 GMS (geostationary meteorological satellite) satellite visible pictures (covering the range ª5000 km down to 5 km centered over the Pacific Ocean). On a log-log plot, the spectrum is very nearly linear, indicating that the scaling is well satisfied. The spectral exponent is nevertheless somewhat smaller (bª1.3) than the advanced very high-resolution radiometer (AVHRR) Florida pictures analyzed in Lovejoy et al. 1993 (bª1.7).

Figure 2b shows the corresponding average spectrum (97 images) over the Oklahoma Cloud and Radiation Testbed (CART) site (280 km to 1.1 km), showing slightly larger b (ª1.5). The discrepancies are likely due to the difference in underlying surfaces (ocean versus land). Note the flattening in both 2a, 2b at the extreme factor ª2; this is due to the inadequate quantization (and hence dynamical range) of the data, which effectively introduces noise and breaks the scaling (see below). Figure 2c investigates any possible seasonal variation; the mean of the available data over eight consecutive months is shown; no significant differences are observed. Similarly, Figure 2d shows the scene-by-scene

(a) International Satellite Cloud Climatology Project

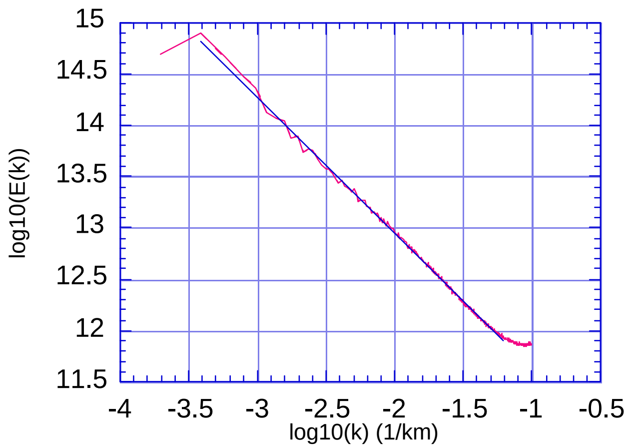

Figure 2a. Average spectrum of 29 GMC images over the Pacific in March and April. This is the average of the power spectra of 29 GMC visible images (0.65 mm) over the western Pacific. The images were taken from 21 March to 10 April 1996, all at 0424 GMT. The images have a resolution of 5.0 km and a dimension of 1024x1024 pixels. The absolute slope of the graph is bª1.32.

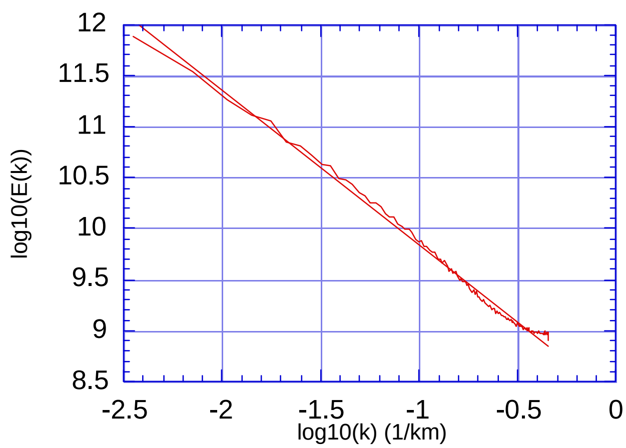

Figure 2b. Average spectrum for 97 NOAA 12 images, channel 1, January to September 1996. This is the average power spectrum of 97 NOAA 12 (AVHRR) images, channel 1 taken from January to September, 1996. The images were all taken between 1330 and 1430, GMT, over the ARM site in Oklahoma. The images were taken using the visible channel 1 (0.58-0.68 mm) with a resolution of 1.1 km. All images are 256x256 pixels. The average bª1.51 used for comparison on other figures is obtained from this graph.

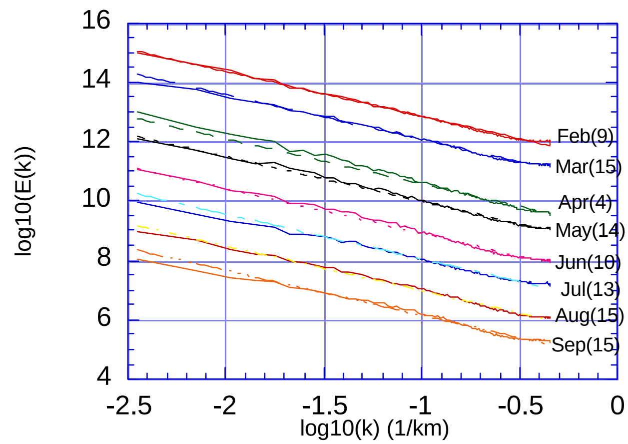

Figure 2c. Monthly averaged spectra for NOAA 12, channel 1. The average spectra for eight months' NOAA 12 (AVHRR) data are shown here. The spectra are all displaced from each other and each month is indicated, along with the number of spectra used in the average, next to the relevant plotted data. Each spectrum is shown for comparison next to a line of the average b over all 97 images used, 1.51.

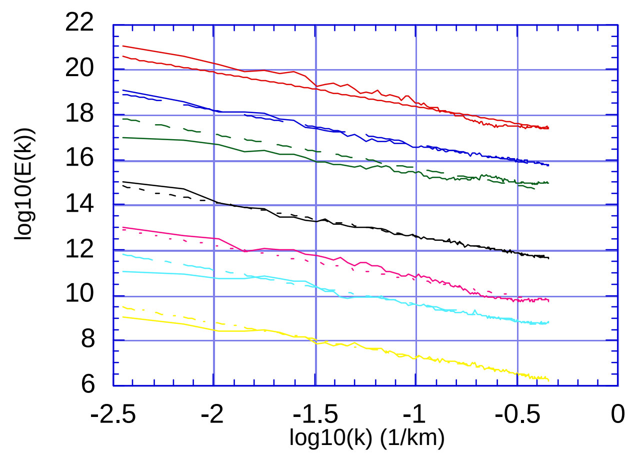

Figure 2d. Every second NOAA 12, channel 1, spectrum for May. The spectra of half of the NOAA 12 (AVHRR) images obtained in May 1996 are shown here displaced from each other and compared with the slope of the average spectrum for all 97 images, bª1.51. From top to bottom, the dates of the images are 1, 3, 8, 11, 15, 21, and 30 May.

variation within a typical month indicating that the day-to-day spectral variability is fairly small. Finally, Figure 2e shows the comparison of the mean geostationary, AVHRR, and the single available SPOT image (20 m resolution), all showing good scaling (down to k=(40 m)$^{-1}$ in the case of SPOT). Note that the SPOT exponent (b^{a}1.9) is somewhat larger. It is not currently known if this is statistically significant (the standard deviation of the NOAA exponents is ±0.2, putting it within two standard deviations of the CART exponent over land, but within one standard deviation of the Florida value).

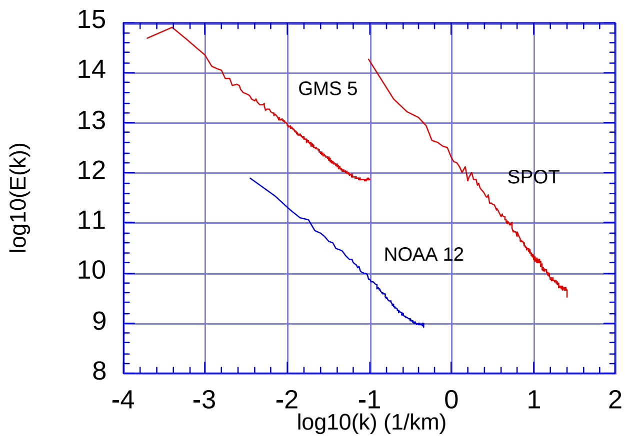

Figure 2e. Comparison of the average GMS, NOAA, and SPOT spectra (right). The average spectra obtained earlier for the GMS, NOAA 12, and SPOT are shown here together for comparison (not artificially displaced). The GMC spectrum shows a range of scales of 5120 km to 10 km, the NOAA 12 spectrum shows a range from 256 km to 2 km, and the SPOT spectrum shows a range from 10 km to 40 m.

Although we have not finished systematic multifractal analysis, some initial results are presented in Figure 3 for the GMS visible data. Recall that in general, scaling systems will be multifractal, characterized by an infinite hierarchy of scaling exponents (each intensity/singularity level will have a different exponent). This infinite number of parameters would render scaling quite unmanageable were it not for the existence of stable, attractive (hence "universal") multifractal processes (e.g., Schertzer and Lovejoy 1997). We confirmed that the radiance fields are indeed very close to what is expected for universal multifractals. In the latter, the infinite hierarchy of exponents (e.g., dimensions/codimensions) is described by only three universal exponents (H, C_1, a): the nonconservation exponent (H) of the mean field; the mean singularity (C_1) and (Levy) index of multifractality (a). Figure 3 shows the

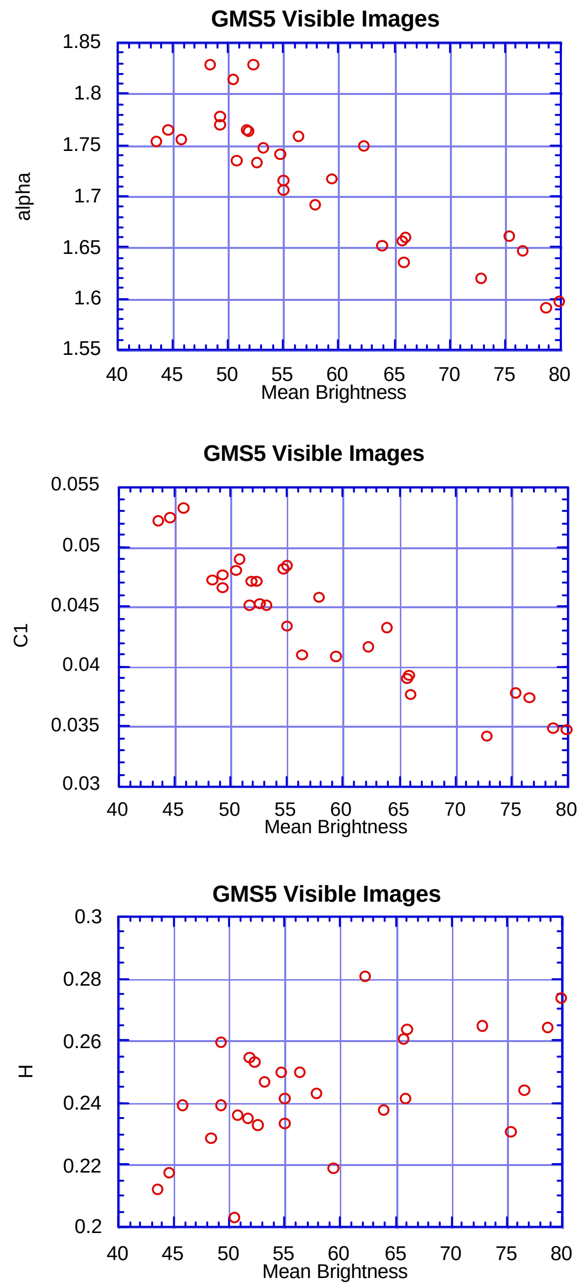

Figure 3. Showing the systematic variation of the multifractal indices a,C_1, H as functions of the mean image brightness (in units of digital counts) for the Pacific Ocean GMC visible data (central 5000 X 5000 km, portion at resolution 5 km). The low brightness cases correspond to small amounts of cloud, and yield C1, a near the values of the surface topography (a^{a}1.8, C_1^{a}0.05; as predicted by simple models for the surface bidirectional reflectance function), which presumably the high brightness end corresponds to the characteristics of clouds without much direct surface influence (a^{a}1.6,C_1^{a}0.03). Roughly the same behavior and values were observed on the CART site AVHRR data. The value of H (which is primarily sensitive to b) seems relatively constant.

relatively small variation in these parameters for the various GMS scenes (at roughly constant sun and viewing angles). The variation seems directly related to the amount of cloud cover as shown by the steady variation of parameters as functions of the average scene brightness.

An Experiment to Determine the Sub AVHHR Resolution Scaling

Although it may seem surprising, there appears to be no systematic study of the scaling of cloud radiances at scales below roughly the resolution of the AVHRR data. The partial exceptions are Cahalan and Snider (1989), who analyzed about 1% of a single LANDSAT picture (down to 30 m); Lovejoy et al. (1993), each of whom analyzed three LANDSAT MSS pictures (160 m resolution); Barker and Davies (1992) and Davis et al. (1997), who each analyzed a single LANDSAT TM image (30 m); and the SPOT analysis (20 m) presented above.

Aside from the nearly prohibitive cost of the necessary large quantities of LANDSAT TM and SPOT data, these sensors were not designed for clouds and are prone to saturation even in the presence of only moderately thick clouds. We therefore decided to take our own photographs, digitize and analyze the results. Aside from the low cost, a further advantage is that resolutions of less than 1 m are readily achieved. The results (including the technical details) will soon be published; suffice it to say that the fundamental limitation is the low (8 bit) dynamical range of available commercial scanners. A simple model for this quantization effect (checked using multifractal simulations) is that it roughly mimics the introduction of a (near) white noise of unit amplitude, breaking the scaling at a scale corresponding to the point where the spectral variance is reduced by $(2^8)^2$ from the low frequency maximum. Hence for example, if b=2 (close to that found here), then the maximum range of scales available before the quantization spoils the variance estimates (and hence the scaling) is ª2^8 (the variance at wavenumber 2^8 will be $(2^8)^2$ times lower; i.e., it will already be at the quantization level).

Figure 4 shows the result for ten (more or less randomly chosen) pictures taken in the Montreal region near local noon (during the period March - July). The resolutions were estimated via knowledge of the cloud base heights and are probably only valid to within ±50%. As can be seen, the scaling is striking over the entire observed range (from larger than 1 km down, in one case of particularly low cloud, to 50 cm).

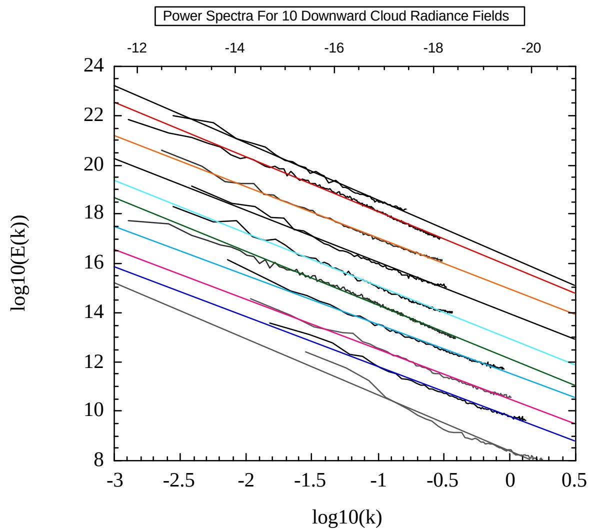

Figure 4. Spectral analysis of ten downward propagating radiation fields, including power law fits for a random collection of clouds over Montreal. The spectra are offset in the vertical for clarity; the resolution was determined by the height of the cloud base. Wavenumber units are in m^{-1}.

The value of b was fairly constant (b=2.1±0.1); close to the SPOT value (1.9), but was larger than the AVHRR and GMS values. This is probably due to the fact that the background (the blue sky) is totally homogeneous and is hence smoother (higher b) than expected for upwelling radiances, which will be affected by surface albedo variability. Full analysis of another 50 or so pictures is currently in progress.

We conclude that the visible radiances are quite remarkably scaling over the entire range of 5000 km to <1 m; it likely continues down to the dissipation scale (<1 cm). The weak evidence for scaling breaks at 60 m (Davis et al. [1997]; presented on the basis of a single factor of 2 on a single LANDSAT image) was not corroborated.

Modeling of Radiative Transfer Through Multifractal Media

Scattering Statistics in Multifractal Clouds

Here we outline recent results that provide the basis for systematic study of radiative transport in multifractal media (see Brosamlen 1994, Lovejoy et al. 1995). In these papers, we indicated how formulae analogous to the multifractal optical density field arise for radiative properties, developing a multifractal scattering formalism in which the (non-dimensional) extinction coefficient k takes the place of the scaling parameter l. For example, the analogue of the scaling exponent of the q^{th} moment of the cloud liquid water (K(q)) with resolution l is $K_p(q_p)$, which describes the scaling of the q_p^{th} moment of the single-photon path distance as a function of k. For multifractal clouds with K(q) analytic at the origin, these results enabled us to relate (to leading order in k>1, ignoring proportionality factors) the multifractal transmission statistics, to those of a homogeneous cloud with an equivalent "effective" extinction coefficient k_{eff}:

$$K_{eff} \approx K^{(1-K'(0))^{-1}} \qquad (1)$$

(note, $K'(0)<0$). Although this result seems highly general (requiring only analyticity), there are theoretical reasons to believe that K(q) may not be analytic; specifically that the LWC is of universal multifractal type characterized by an exponent 0^2a^22 (c.f. the value $a^{a}1.6$ for the radiance field above). Recently we extended the previous work to the cases 1^2a^22; we find (again to leading order) the highly interesting new behavior:

$$K_{eff} \approx (\log K)\frac{\alpha}{2} \qquad (2)$$

This is a much more drastic reduction in the effective extinction coefficient and is associated with (roughly) truncated Log-Levy scattering statistics in which from time to time (due to large "holes"), photons travel extremely far—even in clouds that are on average very optically thick. This effect can be quantified by comparing the most probable distance with the R.M.S. distance (the latter is important in multiple scattering); these quantities are nearly equal in thick clouds with analytic K(q) at q=0, but in these universal multi-fractal clouds, they vary as $K^{-(1+C_1/(\alpha-1))^{-1}}$ and $(\log K)^{-}\frac{\alpha}{2}$, respectively. Note that these occasional long distance scatters will, in the presence of water vapor, significantly contribute to anomalous cloud absorption.

Simulation and Visualization of Transfer Through Three-Dimensional Multifractal Clouds

To generate accurate statistics over the wide ranges of scales and over the large number of realizations necessary, two-dimensional, discrete angle transfer is probably necessary (Lovejoy et al. 1990). However, the ability to simulate more realistic clouds, including realistic modeling of the transfer, is also very important not only for testing the validity of the multifractal model itself, but also for studying many interesting effects. These include the way the directional radiation patterns are affected by the high variability, the cloud appearance, as well as how different sampling/measurement strategies work. The latter point could potentially help resolve the cloud absorption anomaly, which at present requires assumptions of both spatial and angular homogeneity. Figure 5 shows some early results of this project (based on Monte Carlo techniques); the result does indeed remarkably resemble a cloud even though, for the moment, isotropic (rather than stratified) clouds were used, and isotropic (rather than realistic) scattering phase functions were used.

Conclusions

We have reported on the initial results of the first attempt to systematically analyze the scaling properties of statistically significant numbers (several hundred) of cloud images over most of the meteorologically significant range of scales (0.5 - $5X10^6$m). Although, theoretically, scaling is only expected to hold on a statistical ensemble, we found that it held quite accurately even on individual cloud images. Due to the strong (nonlinear) coupling between the cloud radiances and LWC and dynamics, we may infer, in accord with the unified scaling model, that the atmospheric dynamics also satisfy the scaling symmetry. Our results will be very difficult to explain in the standard model involving a meso-scale gap. Our results also imply strong power law resolution dependencies that all radiation budget estimates must explicitly take into account.

It has been known for some time that this strong multifractal heterogeneity leads to relations between albedo and LWC, which involve nonstandard exponents that can lead to arbitrarily large heterogeneity effects if the cloud is thick enough. We developed this, sketching some new theoretical results on scattering in multifractal clouds, as well as an example of the use of 3-dimensional multifractal cloud/radiation models.

Acknowledgments

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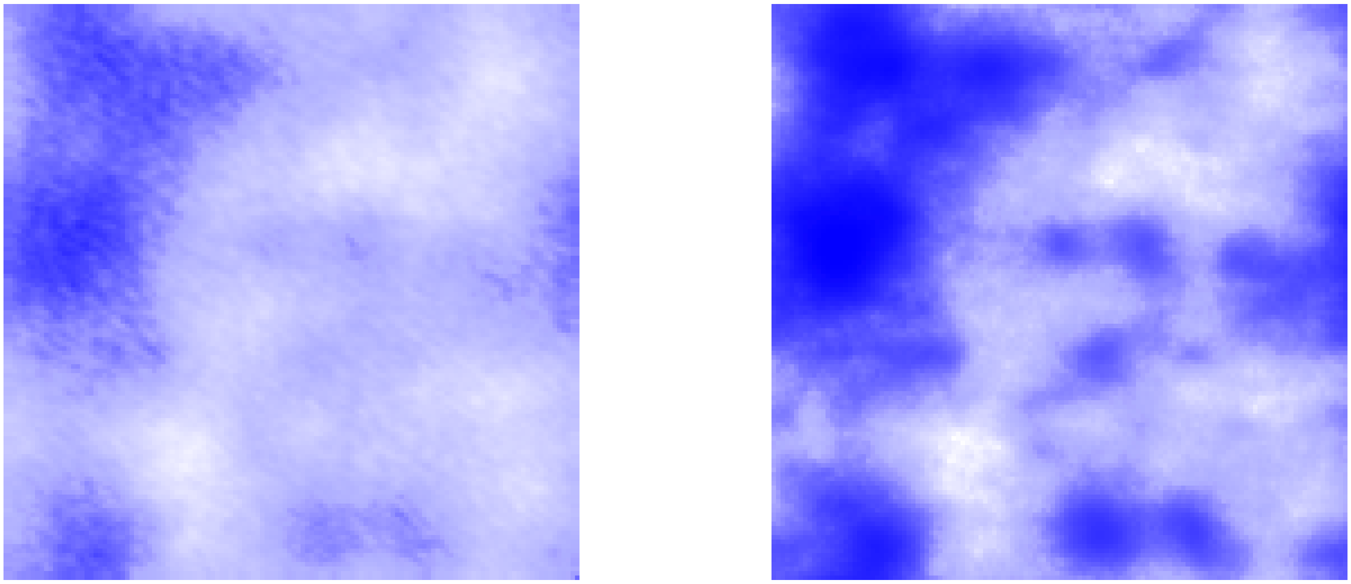

Figure 5. The left-hand column shows the results of the simulation of 200 million photons through a cloud with the mean optical thickness vertical optical thickness = 0.844, and isotropic phase functions. For the simulation, the sun was incident at q (azimuthal angle) = f (polar angle) = 0.1 radians, periodic boundary conditions were used in the horizontal. The cloud was generated with a continuous multifractal cascade process with the universal multifractal parameters of observed satellite radiances: a=1.35,C1=0.15, H=0.3. The simulation was performed on a 128x128x32 grid. The image is generated for an observer at q=15 degrees, f=0. The right hand column shows the optical thickness as a function of space for the angles of the corresponding pictures in the left column. Note that in all cases, the images were normalized by the maximum so as to enhance contrast.

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