Modeling Errors in Diffuse-Sky Radiation

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Most current methods for calculating radiative transfer treat light as a scalar quantity, even though it is well known that a proper description of light requires explicit recognition of its electromagnetic nature. According to the Maxwell equations, the electromagnetic radiation field possesses vector properties and consists of a large number of plane-wave packets with specific phase and polarization. Thus, a light beam is actually a vector quantity described by its Stokes parameters, which are appropriate time averages over the individual plane-waves that express the macroscopic properties of the light beam as a 4-vector quantity,

$$\mathbf{I} = \{I, Q, U, V\}$$

where I is the total beam intensity, and Q,U,V describe the nature of beam polarization in terms of its degree and direction of linear and elliptical polarization. Multiple scattering in a homogeneous, plane-parallel, macroscopically isotropic atmosphere is described by a vector radiative equation of transfer that can be solved numerically with very high precision using the vector doubling/adding method (e.g., Hansen and Travis 1974).

Through the pioneering work of Chandrasekhar (1950), we know that for pure Rayleigh scattering, treating light as a scalar quantity can produce errors as large as 10% in the computed intensity of the radiation field, depending on optical depth and scattering geometry. Nevertheless, because of the significantly greater modeling complexity and computing expense (several orders of magnitude) of the vector doubling/adding calculations compared with scalar doubling/adding, there has been an understandable reluctance to employ the more rigorous method of computation if the simpler treatment is found to be adequate. Errors in reflected intensity because of neglect of polarization were examined by Hansen (1971), who concluded that, in most cases, the errors arising from the scalar approximation are less than about 1% for reflected light by clouds composed of spherical particles with sizes of the order or larger than the incident light. This makes the scalar approximation applicable and practical for radiance calculations for typical cloud and aerosol layers. For climate-related applications, where fluxes and albedos are the radiative quantities required, the integration of radiances over scattering angle has the fortuitous effect of averaging out the radiance errors to the point where no one has seriously worried about the adequacy of the scalar approximation in radiative transfer modeling. Moreover, the nature of radiative modeling in many climate-related applications is in terms of relative differences, or ratios of radiative fluxes, further diluting the significance of any potential errors arising from the scalar approximation. Thus, it would seem that scalar calculations of radiative transfer should be adequate for modeling radiative flux and for comparing model results to measurements.

As part of the overall ARM objective to understand and validate radiative model results and measurements, the original focus of trying to understand the nature of clouds and their impact on atmospheric radiation has now also spread to the clear-sky case (which was supposed to have been a no-problem area). However, recent comparisons of model results to rotating shadowband spectroradiometer, multifilter rotating shadowband radiometer, and pyranometer measurements have demonstrated substantial differences between measured and modeled values of clear-sky diffuse radiation. To what extent these differences may be attributable to inadequate modeling, or to inadequate instrument calibration, is not yet clear. Therefore a closer look at the scalar approximation is warranted in order to put forward the most rigorous theoretical model of radiative transfer for comparison with the observations.

In an earlier study, we examined quantitatively the errors that are induced by the scalar approximation in radiance calculations for the idealized case of Rayleigh-scattering atmospheres (Mishchenko et al. 1994). In agreement with previous studies of this problem, we found intensity errors (both over- and under-estimates) that can be as large as 10% arising in specific geometrical configurations, with the maximum error occurring for optical depths near unity. The addition of a Lambertian reflecting surface tends to dilute the intensity errors. Also, for optically thin atmospheres, the vector/scalar intensity differences are first seen to increase as the single-scattering albedo, ω_o, is reduced from unity (conservative scattering) to about 0.8, but then decrease with further increase in particle absorptivity. Our study also indicated that the error in the scalar approximation arises from low-order (except first-order) light scattering paths that involve right-angle scattering of polarized light with right-angle rotations of the scattering plane that cannot be properly approximated when light is treated as a scalar quantity.

Here we provide additional examples of radiative transfer calculations that further illustrate the inadequacy of the scalar approximation to model the angular distribution of diffuse radiation with the necessary precision and, thus, to serve as the standard of comparison against observational results. The results show that the intensity errors due to the scalar approximation are not symmetrical in azimuth; they vary strongly with the solar zenith angle; and they depend on the nature of the surface reflectivity and on the optical depth and optical properties of atmospheric aerosols.

While simple in concept, the examples we show are rigorous in their treatment of the vector nature of light and of multiple scattering in a plane-parallel, horizontally homogenous atmosphere. The examples are intended to illustrate the nature of the shortfall of the scalar approximation as it pertains to typical model/measurement intercomparisons of diffusely scattered atmospheric radiation. Because the sunlight incident on the atmosphere is unpolarized, it is described by a flux vector $\pi\mathbf{F} = \pi\{1,0,0,0\}$. We compute the bi-directional reflectances $\mathbf{R}(\mu,\mu_o,\varphi-\varphi_o)$, relevant for comparisons to aircraft and space-based measurements, and the diffuse down-welling radiances $\mathbf{D}(\mu,\mu_o,\varphi-\varphi_o)$, which include the multiple scattering contributions between the ground surface and the atmosphere and are relevant to up-looking surface-based measurements. The results are displayed as normalized radiance differences (percent deviations) between the vector and scalar doubling/adding results. In both cases, the computations are made with a sufficient number of quadrature points (15) to eliminate model numerics as a source of error.

Figure 1 shows sky maps of the (Vector-Scalar)/Vector results for diffuse transmission for a pure Rayleigh-scattering atmosphere of optical depth τ_R=1 (relevant for UV wavelengths) with zero surface albedo. The observer is oriented in the $\varphi-\varphi_o=0°$ direction with the sun located behind the observer at the points μ_o= 0.9, 0.5, and 0.1, respectively, which are cosines of the solar zenith angle. In the figure, the horizon is located at μ=0, with the zenith at μ=1. Shaded regions of the sky indicate regions where the scalar approximation underestimates or overestimates the amount of diffuse radiation. The white regions indicate regions where substantial agreement exists (within 1.5%) between the vector and scalar calculations. The shaded regions stretching from the zenith toward $\varphi-\varphi_o$ = 90 and 270° designate those areas where the scalar approximation tends to over-estimate the radiances.

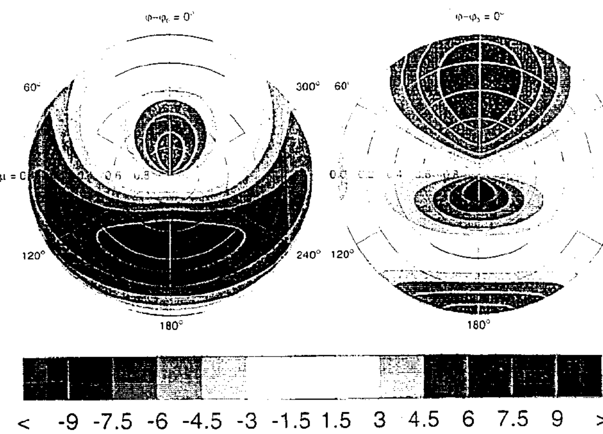

Figure 1. (Vector-Scalar)/Vector diffuse transmission differences for Rayleigh atmosphere with τ_R=1 and no reflecting surface. Left panel results are for high sun with μ_o=0.9. Right panel shows results for μ_o=0.5.

Of particular interest is the systematic change in the characteristic error distribution with the change in solar zenith angle. At high sun, the scalar approximation tends to under-estimate the near-zenith radiances; but at low-μ, the scalar radiances produce over-estimates. As the sun goes lower in the sky, the scalar approximation shifts into over-estimating near-zenith radiances in a characteristic bi-polar pattern which under-estimates diffuse radiances at low-μ in $\varphi-\varphi_o=0°$ scattering plane, but over-estimates them in regions orthogonal to this plane. The maximum error for both over-estimates and under-estimates is about 10%. With respect to diffuse reflectance errors, there is a basic complementarily in the patterns (opposite sign) for the reflected radiance errors, especially for small optical depths, which holds for high to solar zenith angles.

This basic complementary in the reflection/transmission error pattern shape tends to disappear toward low sun angles, until for values near μ_o=0.1, the vector/scalar diffuse intensity error pattern for diffuse reflection tends to resemble that for diffuse transmission, but with a somewhat weaker amplitude in the error pattern (see Figure 2).

Generally speaking, decreasing the Rayleigh optical depth of the atmosphere, as well as increasing the albedo of an underlying Lambertian surface will retain the basic error pattern exhibited in Figure 1, but with a decreased amplitude for the radiance errors. The situation becomes more complicated in the case of an underlying ocean surface where light that is reflected near the Brewster angle produces a

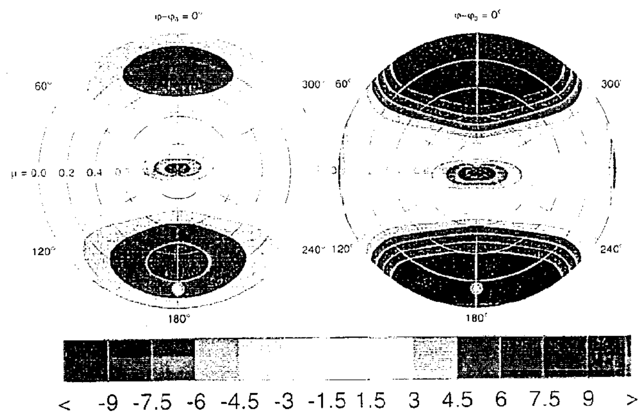

Figure 2. (Vector-Scalar)/Vector diffuse reflection (left panel) and diffuse transmission differences (right panel) for Rayleigh atmosphere with τ_R=1 and no reflecting surface. The results are for low sun with μ_o=0.1.

polarization pattern that is significantly different from the Rayleigh atmosphere pattern. Because of this, a substantially different pattern for second order scattering is established (between the atmosphere and the ocean), which then results in the characteristic 'face mask' pattern for reflected radiation errors that shown in the left-hand panel of Figure 3. This feature is not present in the error pattern for diffusely transmitted light. The results are not particularly sensitive to wind speed, for which a value of 7.2 m/sec was used. The effect of sun glint is evident in the left-hand panel (but not in the right-hand panel), where unpolarized light is reflected in

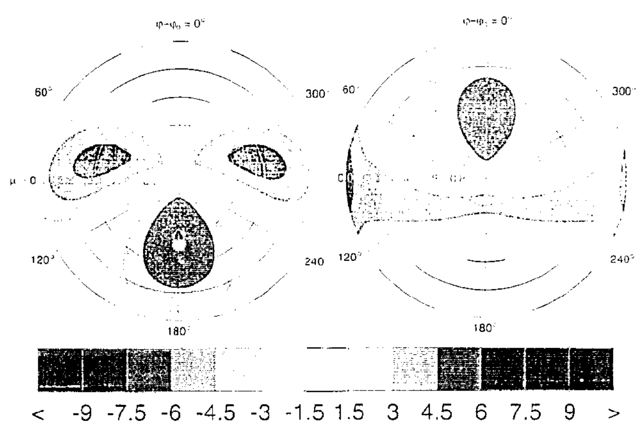

Figure 3. (Vector-Scalar)/Vector diffuse transmission (right panel) and diffuse reflection (left panel) difference for Rayleigh atmosphere with τ_R=0.2 and μ_o=0.5 over an ocean surface with waves for 7.2 m/sec wind speed.

the forward direction to create a substantial broadening in the swath of white area where the vector and scalar results show good agreement.

Atmospheric aerosols, with their own characteristic polarization patterns depending on their size, shape, and refractive index, can also distort the vector/scalar error distribution relative to the basic Rayleigh/Lambert pattern. For moderate particle sizes, however, aerosols tend mainly to dilute the vector/scalar error difference in a fashion similar to that for increasing the Lambertian surface albedo. For cases with heavy aerosol loading with optical depth near unity, the overall intensity errors tend to be of the order of 1%. This result is in basic agreement with Hansen's conclusions (1971) that for particles larger than the incident wavelength, even though they exhibit strong polarization signatures at the rainbow angles, the relative magnitude of the vector/scalar diffuse intensity error is greatly diminished compared with that for Rayleigh scattering.

In the process of calculating the clear-sky diffuse fluxes, integrals are performed over both the φ-φ_o and μ angle directions with the result that most, but not all, of the intensity errors introduced by the scalar approximation are averaged out. This, of course, is extremely fortunate and is the main reason behind the continued success of radiative transfer modeling that uses an approximation that is not really justifiable on physical grounds. Nevertheless, the fact that the magnitude of the errors can be as large as 1% in albedo and in plane transmission is not at all reassuring, particularly when we are attempting to validate radiative model results against observational results to within a few W/m^2.

The degree of cancellation of errors in diffuse flux and albedo that arise from the scalar approximation appears to be most effective for intermediate values μ_o. This is illustrated in Figure 4 below for plane albedo computed as a function of Rayleigh optical depth and solar zenith angle over a wind-roughened ocean surface.

The above results provide compelling evidence that the scalar approximation for light propagation, though widely used in a variety of different radiative transfer methods, is intrinsically deficient and, consequently, inadequate for those situations that require a high degree of precision for measurement analysis. It is also clear from the rather complicated dependence of the radiance errors on scattering geometry, ground surface and atmospheric constituent properties, that simple scaling corrections will not solve the problem. The scalar approximation will undoubtedly continue to be widely used for a wide range of modeling applications. In

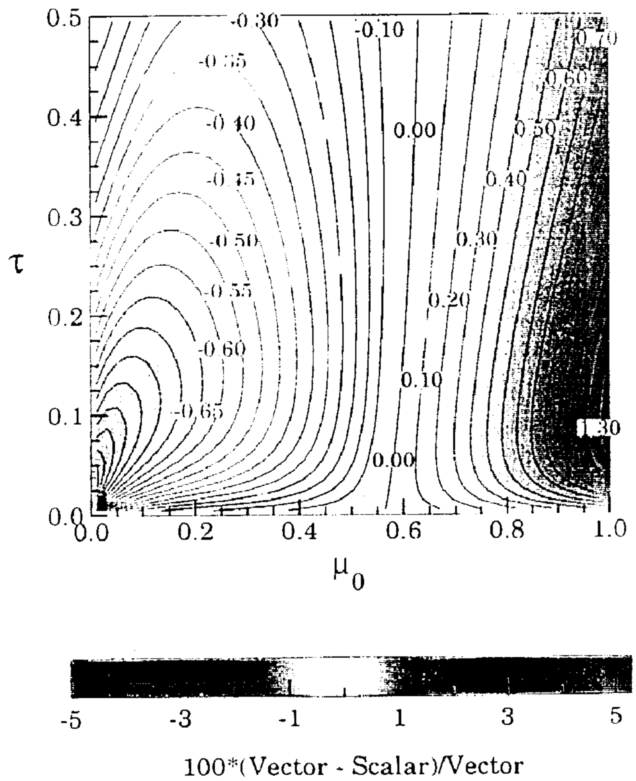

Figure 4. (Vector-Scalar)/Vector diffuse reflectance (plane albedo) differences for a Rayleigh atmosphere over an ocean surface as a function of τ_R and μ_o.

situations where high-precision modeling is required, however, there is really no substitute for doing radiative transfer calculations except the vector doubling/adding way.

The results are equally applicable to questions related to instrument calibration, particularly to instruments used to measure atmospheric fluxes, especially under 'clear-sky' conditions. This is because most of the vector/scalar error is contributed by Rayleigh scattered light, which is strongly polarized, with the degree of polarization and sky brightness changing with time and relative position in response to changes in solar zenith angle. Furthermore, because light that is reflected by mirror surfaces is also very strongly polarized, special care is exercised in instrument design to retain azimuthal symmetry so that the polarization effects will cancel out. Nevertheless, even for carefully designed polarimeters, there remains a small residual instrumental polarization that must be calibrated out. Of even more concern are wide field instruments, which require substantial cosine corrections as to whether or not they may exhibit differential sensitivity to polarized light. In such cases, the instrument response to diffuse sky radiation may exhibit biases to varying distributions diffuse-sky polarization, similar to the vector/scalar biases that have been demonstrated to exist in typical radiative transfer model generated results. One simple test to check instrument sensitivity to variations in sky-light polarization would be to rotate the instrument to see if its response remains azimuthally invariant.

References

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