Fractional Integration and Radiative Transfer in a Multifractal Atmosphere

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Recently, Cess et al. (1995) and Ramathan et al. (1995) cited observations which exhibit an anomalous absorption of cloudy skies in comparison with the values predicted by usual models (homogeneous atmosphere) and which thus introduce large uncertainties for climate change assessments.

These observations raise questions concerning the way general circulation models (GCMs) have been tuned for decades, relying on classical methods, of both radiative transfer and dynamical modeling, limited to studying the radiation/dynamics relationship on an arbitrary scale (often considered as "characteristic"), although the interactions between clouds and radiations occur over a wide range of scales.

These observations also tend to demonstrate that homogeneous models are simply not relevant in relating the highly variable properties of clouds and radiation fields. However smoothed, the intensity of clouds' multi-scattered radiation fields reflect this extreme variability.

Byrne et al. (1995), in order to explain this anomalous absorption, proposed a simple model of broken clouds and measured an increase in the value of the photon mean free path in comparison with the value calculated for a homogeneous atmosphere. The modeled media is a layer filled only with a mixture of clouds and portions of clear skies. They argue that photons, diffused by a first cloud, can circulate horizontally in a "clear sky" region and be reflected in the opposite direction by another cloud and could thus be "trapped" between two clouds (Figure 1). This phenomenon increases significantly the mean free path of photons and thus increases the total absorption of the layer.

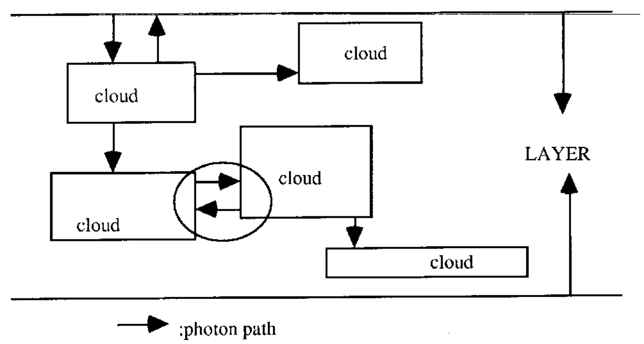

Figure 1. Illustration of the argument of Byrne et al. (1995): Photons are trapped between two or several clouds and thus it increases the global absorption.

However, Byrne et al. (1995) admit that the extreme variability of the radiation fields can be best understood in a multifractal framework (see for instance the different contributions on this subject in Davis et al. 1990, Gabriel et al. 1990, Lovejoy et al. 1990, Schertzer and Lovejoy 1991). Indeed, the (scalar) multifractal model of cloud fields, as discussed below, respects the clouds texture, clustering, bands and intermittency, and the non-linear nature of the true dynamical processes at all scales. This model corresponds to a stochastic model of passive clouds, passively advected by a turbulent velocity field, using coupled cascade processes, and non-linearly conserving the fluxes of energy and concentration variance (Schertzer and Lovejoy 1987). A (scale invariant) multifractal field ϵ_λ is characterized by an infinite hierarchy of singularities γ with associated codimensions $c(\gamma)$; that is, at a scale ratio λ, the probability (Pr) of the fluctuations of the field diverging faster than λ^γ scales as $\lambda^{-c(\gamma)}$ (Schertzer and Lovejoy 1987, 1991).

$$\Pr\left(\varepsilon_\lambda \geq \lambda^\gamma\right) \approx \lambda^{-c(\gamma)} \qquad (1)$$

and the statistical moments scale as

$$\langle \varepsilon_\lambda^q \rangle = \lambda^{K(q)} \tag{2}$$

where $K(q)\ln\lambda$ is the second Laplacian characteristic function (or cumulant generating function) of $\Gamma_\lambda = \ln \epsilon_\lambda$ which is the generator of the process.

The functions $c(\gamma)$ and $K(q)$ are related to each other, using a Legendre transform (Parisi and Frisch 1985) and for "universal multifractals" depend on only three parameters: α (the degree of multifractality), c_1 (the mean homogeneity), and H (the deviation from conservation).

The light propagates through this multifractal cloud density field which respects the Radiative Transfer Equation (Chandrasekhar 1950) and we try to exhibit the relationship between the singularities of the cloud density field and the radiative field with respect to the cloud density. The radiative transfer equation is highly nonlinear and it is not possible to find analytically a straightforward solution.

However it becomes rather simple when applied to a 1D medium. The variation of the radiative field is then proportional to the variation of the optical depth of the cloud (i.e., corresponds to a 1D-integration of the cloud density field). For a plane parallel medium the relationship appears to be a 1D-integration as well. We argue that this could be similar for more general media (i.e., which have inhomogeneities along all directions).

We want then to compare the Radiative Transfer Equation with a fractional integration and therefore we go back to a more general study and determine what happens to the statistics of a fractional-integrated multifractal field.

At first sight, a fractional integration of order H (i.e., a power filter, order H) should only shift the singularities of the fractional integrated field ($\lambda^\gamma \Rightarrow \lambda^{\gamma-H}$).

However it appears that it introduces also a critical singularity below which it is not possible to get any information about the statistics.

We compared two fields, the first one being a multifractal field observed on a given dimension D and its D'-integrated counterpart (D'<D). We studied the relation between their respective moments through the function $K(q)$ and between their codimensions $c(\gamma)$. Then introducing the critical moment $q_{D'}$, due to divergence of high order moments, defined by $K(q_{D'}) = D'(q_{D'}-1)$, we get

$$\begin{aligned} q<q_{D'} &\rightarrow K_{D'}(q) = 0 \\ q>q_{D'} &\rightarrow K_{D'}(q) = K_D(q) - (D-D')(q-1) \end{aligned} \tag{3}$$

For codimensions we have

$$\begin{aligned} \gamma<\gamma_{D'} &\rightarrow c_{D'}(\gamma) = 0 \\ \gamma>\gamma_{D'} &\rightarrow c_{D'}(\gamma) = c_D(\gamma - D + D') - (D-D') \end{aligned} \tag{4}$$

In order to illustrate this point, we made 100 simulations of a multifractal field (Wilson et al. 1991; Pecknold et al. 1993) with α=1.35 (measured for clouds by Tessier et al. 1993), H=0, and C_1=0.75. Then we made a fractional integration of order D'=1 for each realization, and we calculated the moments and codimensions. We represent these on the following graphs which appear to confirm the behavior predicted previously (Figures 2 and 3 where D=2 and D'=1).

Integration smoothes the low order singularities of a multifractal field, which means that it should be also the case for the Radiative Transfer Equation. Thus, the radiance field reflects only the high concentrations of water in atmosphere. However, the bad news is that we have to be very careful with analysis of these fields; they must be fractionally differentiated, otherwise the relevant range of singularities that exhibits the universal exponents C_1, α, and H becomes rather small.

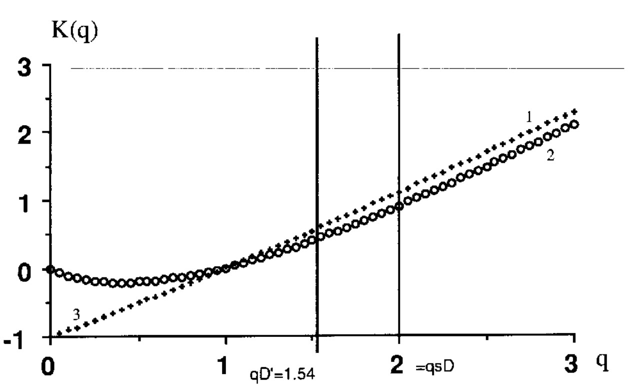

Figure 2. K(q) for D'-integrated and dimension D fields with α = 1.35 and C_i = 0.75 (H=0) for D=2 and D'=1. Captions: + Integrated field on a two dimensional field D' = 1; o D=2.
Region 1: $K_{D'}(q) + q - 1 \approx -1.24 + 1.17q$
Region 2: $K(q) \approx -1.47 + 1.18q$
Region 3: $K_{D'}(q) + q - 1 \approx -1.01 + 0.99q$

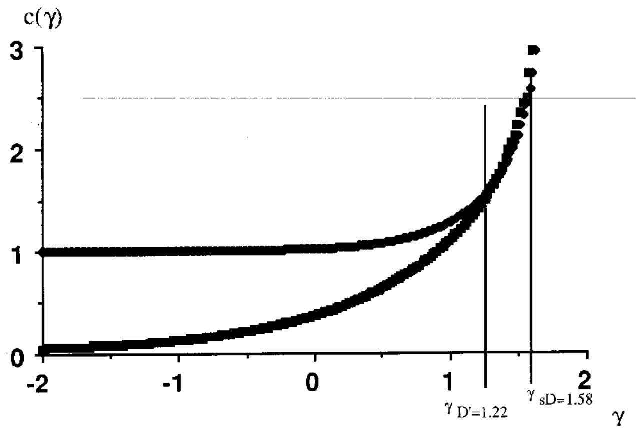

Figure 3. Codimensions of a D dimensional field and the corresponding D'-integrated field with α=1.35 and C_1=0.75 (H=0) for D=2 and D'=1. Captions: ■ D=2; ◆ one dimensional integrated of previous, D'=1.

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