Two Complementary Multifractal Analysis Techniques for Non-Stationary Atmospheric Processes with an Application to Cloud Liquid Water Content

W. Wiscombe and A. Davis
NASA Goddard Space Flight Center
Climate & Radiation Branch
Greenbelt, MD 20771

A. Marshak
SSAI
Lanham, MD 20706

Standard Gaussian-type statistics are not really adequate to describe atmospheric variability; this traditional approach implicitly dismisses fluctuations as "noise" of use only in computing a standard deviation which, together with the mean, presumably varies so little in space and time that one can meaningfully speak of "climatological" values. In geophysics, unfortunately, this approach often fails. Means and standard deviations not only exhibit no "climatology," but depend on the scale of space/time sampling.

We adopt the opposite viewpoint: variability is in fact the signal, not the noise. This variability teaches us about the fundamental physics at work. Even simple laboratory or computational systems (and *a fortiori* geophysical systems) are typically attracted into dynamical equilibria characterized by a large range of length- and/or time-scales, power-law energy spectra, and fractal geometrical properties. For such systems, the concepts of scale-invariance and/or multifractality provide the most productive framework for data analysis and simulation.

Typical atmospheric signals exhibit non-stationary behavior. Our first task is therefore to define the most interesting *stationary* aspects of such non-stationary datasets. One method for doing this, called "Singular Measures," focuses on the absolute values of the *gradient* field, which is more likely to be stationary; as we progressively degrade the resolution r of this field, taking powers q and averaging, the results appear as power-laws in terms of the resolution r where the exponent is a function of q. Another method, called "Structure Functions," focuses on the absolute values of the differences that occur in the data over arbitrarily large or small scales. For detailed descriptions of both methods, we refer to Davis et al. (1993a) for a graphically based tutorial and to Davis et al. (1993b and refers therein) for a wavelet-based approach.

Singular Measures are currently attracting more attention than the once more popular Structure Functions. This is largely due to their close connection with multiplicative cascade models, first invented to describe turbulent energy cascades. While there is no completely general connection between the two methods, for a specific physical process or stochastic model, a relation may exist. However, in a typical data analysis situation, we have no inkling whether such a relation exists; in fact, one of our *goals* is to find a statistical connection between the fluctuations of the field and the fluctuations of its gradients. (We proceed by analogy with classical theory, where a complete description of the system calls for both energy [field] and flux [gradient] terms.) We therefore advocate the use of *both* methods of analysis, either independently (Davis et al. 1993a) or jointly (Davis et al. 1993c). Indeed the two techniques reveal quite different and, in many respects, complementary aspects of the data and, hence, of the underlying physical processes.

Structure Functions characterize the non-differentiability or "roughness" of the data, as well as its degree of stationarity. As an example, Figure 1a shows a time-series of cloud liquid water content (LWC) measured during the Atlantic Stratocumulus Transition Experiment (ASTEX)

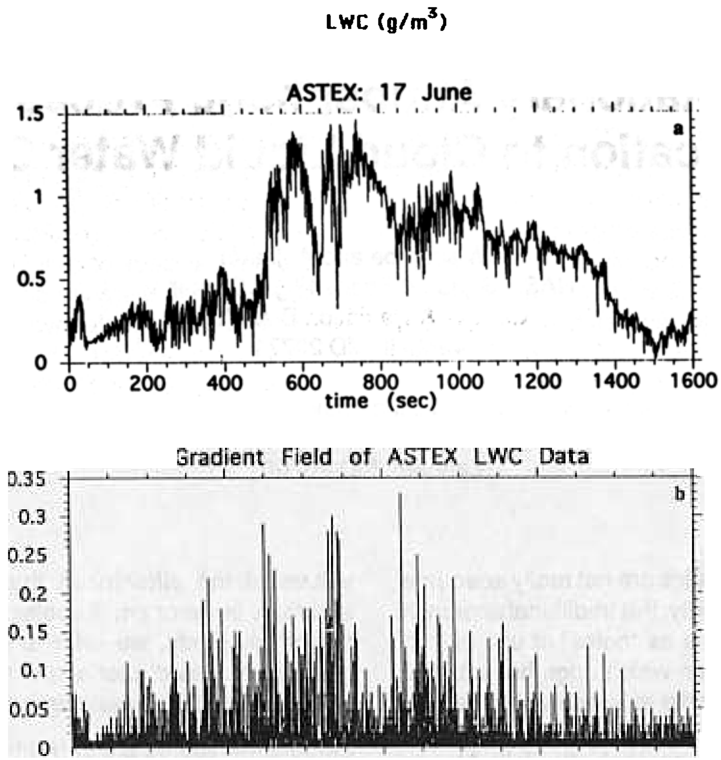

Figure 1. 10 Hz LWC data collected with the Gerber PVM-100 during an ASTEX research flight (Gerber 1992). (a) a segment of LWC data from 17 June 1992. (b) Absolute small scale gradient field of the dataset in (a).

field program in June 1992. The most straightforward measure of the roughness of this graph is its fractal dimension D_g (Mandelbrot 1977): if $D_g = 1$, then the graph is smooth (differentiable), while, at the other extreme, if $D_g = 2$, the graph is so discontinuous that it fills a whole region of two-dimensional space. An alternative quantification of the "smoothness" of the data is $H_1 = 2\text{-}D_g$, which ranges from 0 to 1; at $H_1 = 1/2$, one finds, for example, the case of a one-dimensional random walk. Another, a priori independent, measure of non-differentiability, possibly discontinuity, is the power spectrum exponent (e.g., $\approx -5/3$ for Figure 1a). One can define an infinite number of other exponents, all of which are organized in a "multifractal" hierarchy (Parisi and Frisch 1985).

The Singular Measure method highlights the "intermittency" present in the typically very spiky absolute gradient dataset; see Figure 1b for that associated with the LWC data in Figure 1a. The spikes ("singularities") tend to cluster and to concentrate on sparse subsets of space ("fractals"). Usually many different degrees of singularity are observed in the gradient data, indicating the presence of a multifractal field, so, here too, an infinite number of exponents are needed to describe the data statistically. The simplest of these exponents, the "information" dimension D_1 (e.g., Hentschel and Procaccia 1983), measures the size of the set occupied by those singularities that contribute most significantly to the mean (of the absolute gradients). If $D_1 = 1$, this set has the same dimension as the full time interval and there is no intermittency; if $D_1 = 0$, this set consists of only a finite number of points (including the case where the gradient field is reduced to a Dirac δ-function). Following Schertzer and Lovejoy (1987), it is convenient to use $C_1 = 1\text{-}D_1$, which ranges from 0 to 1 as the intermittency of the gradient field increases from none to the δ-function case.

In Figure 2, we summarize our ideas using regions in a plane. The axes correspond to our two fundamental exponents: horizontally the smoothness parameter H_1, and vertically the intermittency parameter C_1. We call this the "q = 1 multifractal plane" since for starters we are dealing only with first-order statistical moments. We have indicated the loci of various models found in the literature and of various LWC datasets that we have analyzed. Along the vertical axis, we find multiplicative cascade models which are discontinuous and stationary, hence $H_1 = 0$. Along the horizontal axis, we find stationary Gaussian noises (at $H_1 = 0$), Mandelbrot's (1977) additive models known as "fractional" Brownian motions (at $0<H_1<1$) that generalize standard Brownian motion found at $H_1 = 1/2$, and, finally, differentiable functions typical of dynamical models (at $H_1 = 1$). All of these have non-intermittent Gaussian or even bounded gradient fields, hence $C_1 = 0$. Four different LWC datasets are represented: the ASTEX one in Figure 1a and three from the 1987 First ISCCP[a] Regional Experiment (FIRE) marine stratocumulus experiment.

These LWC datasets typically have small but non-trivial C_1's and $H_1 \approx 1/3$ (see Marshak et al. [1993b] for further details). This suggests modeling the LWC data using a class of "hybrid" stochastic models having both multiplicative and additive ingredients in order to give non-vanishing projections on both axes in Figure 2.[b] Schertzer and Lovejoy (1987) invented one such model involving a low-pass power-law filtering (also called a "fractional integration") of a singular multiplicative cascade model. Cahalan et al. (1989) invented another such model, involving smoothing a multiplicative cascade by making the dispersion of the weights decrease with scale; this results in a *bounded* cascade process. All the models indicated on the figure have power spectrum exponent -5/3, as do the empirical data (Cahalan and Snider 1989). Bounded cascade models have very small, essentially residual C_1-values (Marshak et al. 1993a), as does one of the datasets. (By "residual" we mean that, because of the finite range of scales involved, we cannot distinguish them statistically from purely additive models having $C_1 = 0$, according to a simple criterion developed by Davis et al. [1993d].)

(a) International Satellite Cloud Climatology Project.

(b) In the physics and turbulence communities, such models are known as "multi-affine" (Viscek and Barabasi 1991).

In conclusion, we aim to characterize the 3D structure of clouds (as sampled by 1D transects of LWC) using extensions of simple, robust, and turbulence-proven multifractal statistical methods and models. We are aiming for the simplest possible stochastic models with the minimum number of free parameters, yet one capable of capturing variability on the widest possible range of scales. This is obviously an important prelude to the theoretical and empirical study of cloud radiation problems. Beyond that, we believe there will be fruitful applications of these general-purpose scale-invariant data analysis methods for many other Atmospheric Radiation Measurement (ARM) Program purposes: e.g., comparisons of model output with ARM observations at a much deeper level than traditional mean-variance; and interpolations and extrapolations in space and time scale, to mention just a few possibilities.

References

Cahalan, R. F., and J. B. Snider. 1989. Marine stratocumulus structure. *Remote Sens. Environ.* **28**:95-107.

Cahalan, R. F., M. Nestler, W. Ridgeway, W. J. Wiscombe, and T. Bell. 1989. Marine stratocumulus spatial structure. *Proc. 4th Int. Meeting on Statistical Climatology*, pp. 19-25.

Davis, A., A. Marshak, W. Wiscombe, and R. Cahalan. 1993a. Multifractal Characterizations of Non-stationarity and Intermittency in Geophysical Fields, Observed, Retrieved or Simulated. *J. Geophys. Res.* (submitted).

Davis, A., A. Marshak, and W. Wiscombe. 1993b. Wavelet-based multifractal analysis of non-stationary and/or intermittent geophysical signals—To appear in *Applications of Wavelet Transforms in Geophysics*, Foufoula-Georgion and P. Kumar. Academic Press.

Davis, A., A. Marshak, and W. Wiscombe. 1993c. Joint Bi-multifractal analysis and multi-affine modeling of non-stationary geophysical processes, application to turbulence and clouds. *Fractals* (accepted probability distributions for the gradients and the increments of non-stationary processes (In preparation).

Davis, A., A. Marshak, W. Wiscombe, and R. Cahalan. 1993d. A simple test for the hypothesis of multiplicity in scaling of statistical properties in geophysical signals. (In preparation).

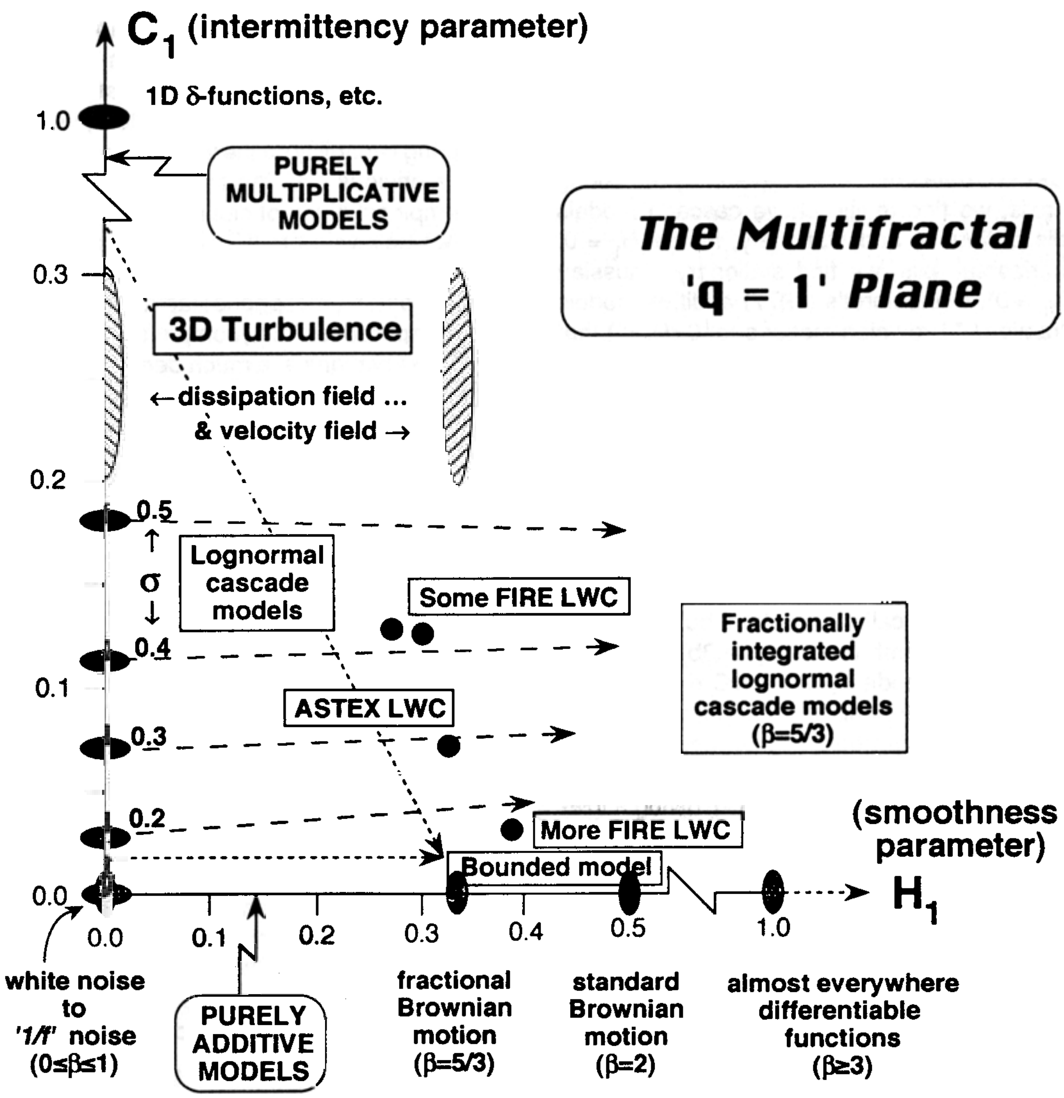

Figure 2. Multiplicative and additive models populate the axes; whereas, "hybrid" or "multi-affine" models populate the plane itself. This new class of models contains, as a simple example, randomly positioned Heaviside steps; they are almost everywhere differentiable C_1 = 3D1, but their gradients are δ-functions (C_1 = 3D1). The arrows indicate the effect of "turning on" the smoothing parameter in two other hybrid models discussed in the text. Note that turbulent velocity or passive scalar transects typically have about the same H_1 but a higher C_1 than we find for our LWC data.

Gerber, H. 1992. New microphysics sensor for aircraft use. 1992. Preprint Volume, *Proc. 11th. Intern. Conf. on Clouds and Precipitation*, Montreal, Canada, Aug. 17-21, 1992, pp. 942-944.

Hentschel, H.G.E., and I. Procaccia. 1983. The infinite number of generalized dimensions of fractals and strange attractors. *Physica D.* **8**:435-444.

Mandelbrot, B. B. 1977. Fractals: form, chance, and dimension. W. H. Freeman and Company, San Francisco.

Marshak, A., A. Davis, R. Cahalan, and W. Wiscombe. 1993a. Bounded cascades as non-stationary multifractals. *Physical Review E* (in press).

Marshak, A., A. Davis, W. Wiscombe, and R. Cahalan. 1993b. The scale-invariant structure of marine stratocumulus deduced from observed liquid water distributions; Part 1: Spectral properties and stationarity issues; Part 2: Multifractal properties and model validation. *J. Atmos. Sci.*, submitted.

Parisi, G., and U. Frisch. 1985. A multifractal model of intermittency. *Turbulence and predictability in geophysical fluid dynamics and climate dynamics,* eds. M. Ghil, R. Benzi, and G. Parisi, pp. 84-88. North-Holland, Amsterdam.

Schertzer, D., and S. Lovejoy. 1987. Physical modeling and analysis of rain clouds by anisotropic scaling multiplicative processes. *J. Geophys. Res.* **92**, (D8):9693-9714.

Viscek, T., and A.-L. Barabasi. 1991. Multi-affine model for the velocity distribution in fully turbulent flows. *J. Phys. A: Math. Gen.* **24**:L845-L851.